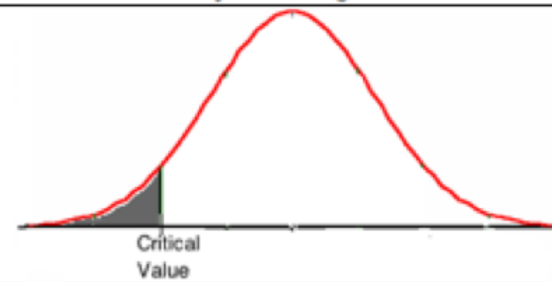
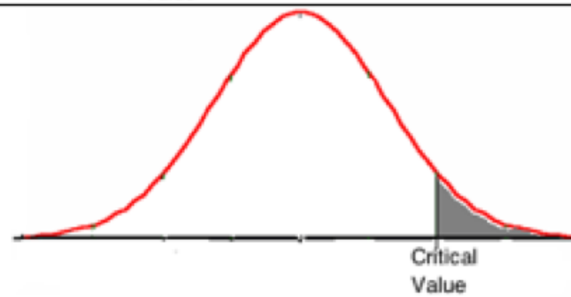
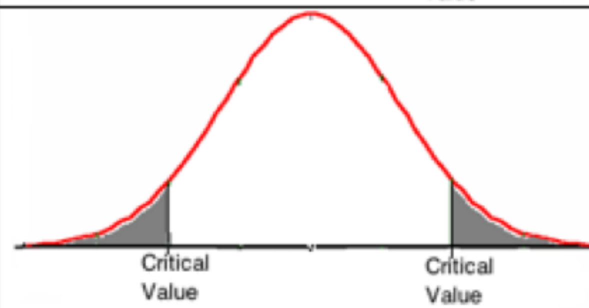
Inference for the Mean of a Population

- In statistics, the term **significant** means that a result is not due to chance.
- A **significance test** is used to address the question, "what is the probability that we found certain results from a sample given the claimed population values" or "is this association between variables due to chance".

- When conducting a significance test, we begin by stating our **null hypothesis** and our **alternate hypothesis**.
- H_0: is the **null hypothesis.**
  - The **null hypothesis** states that there is no effect or change in the population.
  - It is the statement being tested in a test of significance.
- H_a: is the **alternate hypothesis**.
  - The **alternative hypothesis** describes the effect we suspect is true, in other words, it is the alternative to the "no effect" of the null hypothesis.

8.1 Inference for the Mean

- For inference about a population mean:
- H_0: $\mu = \mu_0$
  - where μ_0 represents the given population mean.
- The alternative hypothesis will be one of the following:

Alternate Hypothesis	Rejection Region
$H_a : \mu < \mu_0$	
$H_a : \mu > \mu_0$	
$H_a : \mu \neq \mu_0$	

- The probability, computed assuming that the null hypothesis is true, that the test statistic would take a value as extreme or more extreme than that actually observed is called the ***p*-value** of the test.
- A result with a small *p*-value is called **statistically significant**.
- This means that chance alone would rarely produce so extreme a result.
- We say that a value is **statistically significant** when the *p*-value is as small as, or smaller than, the given significance level, α. If we are not given α, we can interpret the results like this:
  - If the *p*-value is less than 1%, we say that there is overwhelming evidence to infer that the alternative hypothesis is true. (We also say that the test is highly significant)
  - If the *p*-value is between 1% and 5%, we say that there is strong evidence to infer that the alternative hypothesis is true. (We also say that the test is significant)
  - If the *p*-value is between 5% and 10%, we say that there is weak evidence to infer that the alternative hypothesis is true. (We also say that the test not statistically significant)
  - If the *p*-value is exceeds 10%, we say that there is no evidence to infer that the alternative hypothesis is true.

- When performing a significance test, we follow these steps:
  1. Check assumptions.
  2. State the null and alternate hypotheses.
  3. Graph the rejection region, labeling the critical values.
  4. Calculate the test statistic.
  5. Find the *p*-value. If this answer is less than the significance level, α, we can reject the null hypothesis in favor of the alternate.
  6. Give your conclusion using the context of the problem. When stating the conclusion you can give results with a confidence of $(1-\alpha)(100)\%$.

- We will see two types of tests when working with a test about a population mean.
  - One sample mean z-test
  - One sample mean t-test

- z **– test** Assumptions:
  1. An SRS of size n from the population.
  2. Known population standard deviation, σ.
  3. Either a normal population or a large sample ($n \geq 30$).
- To compute the z – test statistic, we use the formula:

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

- ***t* – test** Assumptions:
  1. An SRS of size n from the population.
  2. Unknown population standard deviation.
  3. Either a normal population or a large sample ($n \geq 30$).
- To compute the t – test statistic, we use the formula:

$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

where s is the sample standard deviation.
The t – test will use $n - 1$ degrees of freedom.

Example:

$\mu_0 = 200$ $\sigma = 9$

1. Mr. Murphy is an avid golfer. Suppose he has been using the same golf clubs for quite some time. [Based on this experience, he knows that his average distance when hitting a ball with his current driver (the longest-hitting club) under ideal conditions is 200 yards with a standard deviation of 9.] After some preliminary swings with a new driver, he obtained the following sample of driving distances:

205	198	220	210	194	201	213	191	211	203

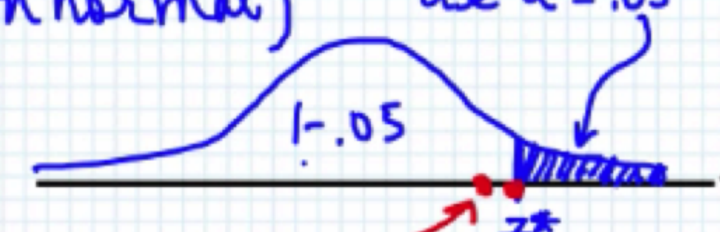
← sample data

Assuming the distribution of distances is normal, do you think the new clubs do a better job? (assume population normal) use $\alpha = .05$

<z-test> $\bar{x} = 204.6$

$H_0: \mu = 200$

$H_a: \mu > 200$



$z^* = \text{invNorm}(.95) = 1.645$

test statistic: $z = \dfrac{204.6 - 200}{9/\sqrt{10}} = \boxed{1.616}$

p value: $P(z > 1.616) = .0530 > .05$ Fail to reject H_0

normalcdf(1.616, 10^6, 0, 1)

Example:

$\mu_0 = 325.16$ $\bar{X} = 312.34$ $n = 75$ $S = 76.42$

2. An association of college bookstores reported that the average amount of money spent by students on textbooks for the Fall 2010 semester was \$325.16. A random sample of 75 students at the local campus of the state university indicated an average bill for textbooks for the semester in question to be \$312.34 with a standard deviation of \$76.42. Do these data provide significant evidence that the actual average bill is different from the \$325.16 reported? Test at the 1% significance level.

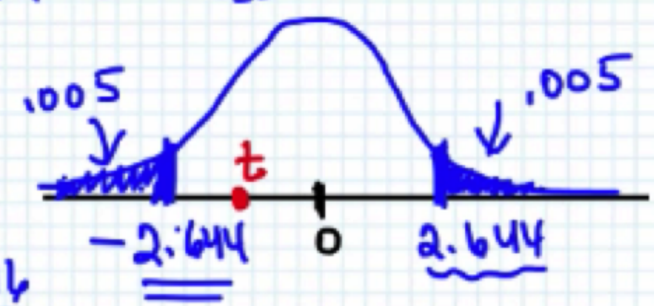
df = 74

$\alpha = .01$

One sample t - test

$H_0: \mu = 325.16$

$H_a: \mu \neq 325.14$



t distribution

invT(.005, 74)

qt(.005, 74)

.995

test statistic $t = \frac{312.34 - 324.16}{76.42/\sqrt{75}}$

$t = -1.4528$

pvalue: $p(t < -1.4528)$ or $p(t > 1.4528) = 2p(t < -1.4528)$

2*tcdf(small, -1.4528, 74) or 2·pt(-1.4528, 74)

pvalue = .1505 > .01 ⇒ Fail to reject H_0

- **Matched pairs *t* – test** is a special test when we are comparing corresponding values in data.
- This test is used only when our data samples are DEPENDENT upon one another (like before and after results).
- Matched pairs *t* – test assumptions:
  1. Each sample is an SRS of size n from the same population.
  2. The test is conducted on paired data (the samples are NOT independent).
  3. Unknown population standard deviation.
  4. Either a normal population or large samples ($n \geq 30$).

Example:

3. A new law has been passed giving city police greater powers in apprehending suspected criminals. For six neighborhoods, the numbers of reported crimes one year before and one year after the new law are shown. Does this indicate that the number of reported crimes have dropped?

(Assume the population is normally distributed)

$n = 6$ $df = 5$

Neighborhood	1	2	3	4	5	6
Before	18	35	44	28	22	37
After	21	23	30	19	24	29
difference (before − after)	−3	12	14	9	−2	8

$\alpha = .05$

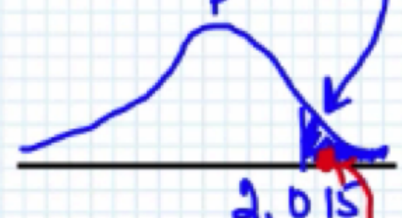
$\bar{x}_D = 6.\bar{3}$

$s_D = 7.174$

$H_0: \mu_D = 0$ (no change)

$H_a: \mu_D > 0$ (dropped crime rate ⇒ before − after > 0)

$$t = \frac{6.333 - 0}{7.174/\sqrt{6}} = 2.16245$$



pvalue $= P(t > 2.16245) =$ tcdf(2.16245, 10^6, 5) / 1−pt(2.16245, 5) $= .0415 < .05$