Comparing Two Means

- **Two – sample mean *t* – tests** compare the responses to two treatments or characteristics of two populations.
- There is a separate sample from each treatment or population.
- These tests are quite different than the matched pairs *t* – test.

- The null and alternate hypotheses would be:

$$
\begin{array}{l} H_0 : \mu_1 = \mu_2 \\ H_a : \mu_1 > \mu_2 \end{array} \quad or \quad \begin{array}{l} H_0 : \mu_1 = \mu_2 \\ H_a : \mu_1 < \mu_2 \end{array} \quad or \quad \begin{array}{l} H_0 : \mu_1 = \mu_2 \\ H_a : \mu_1 \neq \mu_2 \end{array}
$$

- The assumptions for a two-sample mean t – test are:
  1. We have two independent SRSs, from two distinct populations and we measure the same variable for both samples.
  2. Both populations are normally distributed with unknown means and standard deviations. (Or if each given sample size is greater than or equal to 30.)

- Two-sample mean t – test statistic:

= 0 if $H_0: \mu_1 = \mu_2$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

- The degrees of freedom is equal to the smaller of $n_1 - 1$ or $n_2 - 1$.

Example:

p-value: $2P(t > .7827) = .437 > \alpha$
Fail to reject H_0

1. The president of an all-female school stated in an interview that she was sure that the students at her school studied more, on average, than the students at a neighboring all-male school. The president of the all-male school responded that he thought the mean study time for each student body was undoubtedly about the same and suggested that a study be undertaken to clear up the controversy. Accordingly, independent samples were taken at the two schools with the following results:

School	Sample Size	Mean Study Time (hrs)	Standard deviation (hrs)
All Female ①	65. n_1	18.56 $\bar{x}_1$	4.35 s_1
All Male ②	75 n_2	17.95 $\bar{x}_2$	4.87 s_2

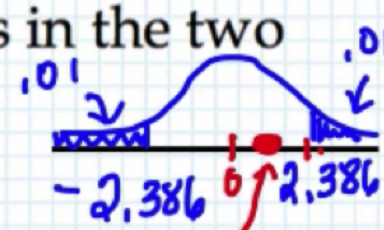
$\alpha = .02$ $df = 64$

Determine, at the 2% level of significance, if there is a significant difference between the mean studying times of the students in the two schools based on these samples.

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \neq \mu_2$

$$t = \frac{(18.56 - 17.95) - 0}{\sqrt{\frac{4.35^2}{65} + \frac{4.87^2}{72}}} = .7827$$



Example:

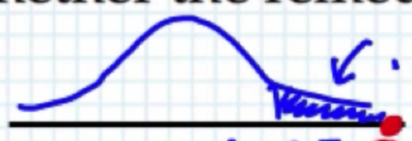
$\alpha = .02$

2. Determine, at the 2% level of significance, if there is a significant difference between the mean studying times of the students in the two schools based on these samples.

	Remedial (1)	Non-remedial (2)
Sample size	100 n_1	40 n_2
Mean Exam Grade	83.0 $\bar{x}_1$	76.5 $\bar{x}_2$
Std Dev for Exam	2.76 s_1	4.11 s_2

$df = 39$

Test, at the 2% level, whether the remediation helped the students to be more successful.



invT or qt(.98, 39)

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 > \mu_2$

$$t = \frac{(83 - 76.5) - (0)}{\sqrt{\frac{2.76^2}{100} + \frac{4.11^2}{40}}} = 9.206$$

pvalue: $p(t > 9.206) = 1.26 \times 10^{-11} \approx 0 < \alpha$

(tcdf(9.206, 10^6, 39)

1 − pt(9.206, 39))

Reject H_0