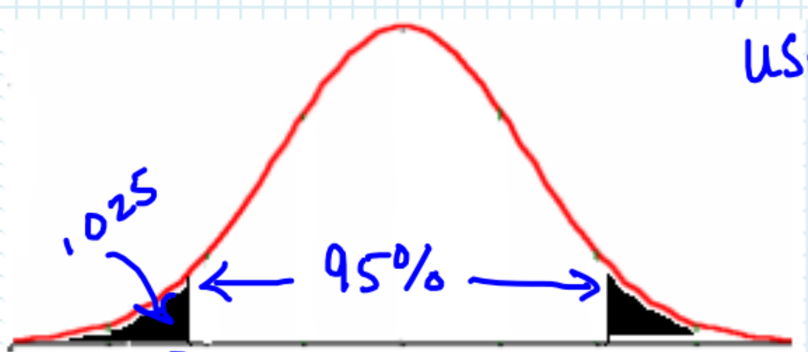
Confidence Interval for a Proportion

- Before any inferences can be made about a proportion, certain conditions must be satisfied:
  1. The sample must be an SRS from the population of interest.
  2. The population must be at least 10 times the size of the sample.
  3. The number of successes and the number of failures must each be at least 10. $np \geq 10$ $n(1-p) \geq 10$
- The sample statistic for a population proportion is $\hat{p}$, so based on the formula for a CI, we have $\hat{p} \pm$ margin of error.

- How do we find the margin of error if it is not given to us?
- The margin of error is equal to the **critical value** (a number based on our level of confidence) and the **standard deviation** (or **standard error** when needed) of the statistic.
- **Critical Value**: When the distribution is assumed to be normal, our critical value is found from the z table (or using invNorm on calculator or qnorm in R). If it is not normal, we will use the t distribution (discussed later).
- **Standard Deviation/Error**: When working with proportions, the standard deviation of the statistic $\hat{p}$ is $\sqrt{p(1-p)/n}$. Since p is unknown, we will use the standard error. To calculate the standard error of , use the formula $\sqrt{\hat{p}(1-\hat{p})/n}$.

- Finding the critical z value for confidence level CL:
- In R-Studio,
  - qnorm(CL + (1-CL)/2)
- With a TI-83/84 calculator,
  - invNorm(CL + (1-CL)/2)

example
for 95% CI



together
= 5% = .05

95% conf. level
use invNorm(.975)
for TI calculator
or qnorm(.975)
for R.

Example:

1. In the first eight games of this year's basketball season, Lenny made 25 free throws in 40 attempts.

   a. What is , Lenny's sample proportion of successes?

   b. Find and interpret the 90% confidence interval for Lenny's proportion of free-throw success.

a) $\hat{p} = 25/40 = .625$

b) $90\% \Rightarrow CL = .90$

critical value: $z^* = \text{invNorm}\left(.9 + \frac{(1-.9)}{2}\right) = \text{invNorm}(.95)$

$z^* = 1.645$

$SE = \sqrt{\frac{.625(1-.625)}{40}} = .0765$

margin of error

$(1.645)(.0765) = .1259$

confidence interval: $.625 \pm .1259$

$= [.499, .751]$

- Sometimes we are asked to find the minimum sample size needed to produce a particular margin of error given a certain confidence level.
- When working with a one-sample proportion, we can use the formula:

$$ME = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Example:

2. It is believed that 35% of all voters favor a particular candidate. How large of a simple random sample is required so that the margin of error of the estimate of the percentage of all voters in favor is no more than 3% at the 95% confidence level?

SE : $\sqrt{\frac{.35(1-.35)}{n}}$

$z^* = \text{invNorm}\left(.95 + \frac{(1-.95)}{2}\right)$

$= \text{invNorm}(.975) = 1.96$

error ≥ formula for M.E.

$.03 \geq 1.96\sqrt{\frac{.35(.65)}{n}}$

$\left(\frac{.03}{1.96}\right)^2 \geq \left(\sqrt{\frac{.35(.65)}{n}}\right)^2$

$.000234 \geq .2275/n$

$n \geq .2275/.00234$

$n \geq 971.07$

$\boxed{n = 972}$

(always round up)