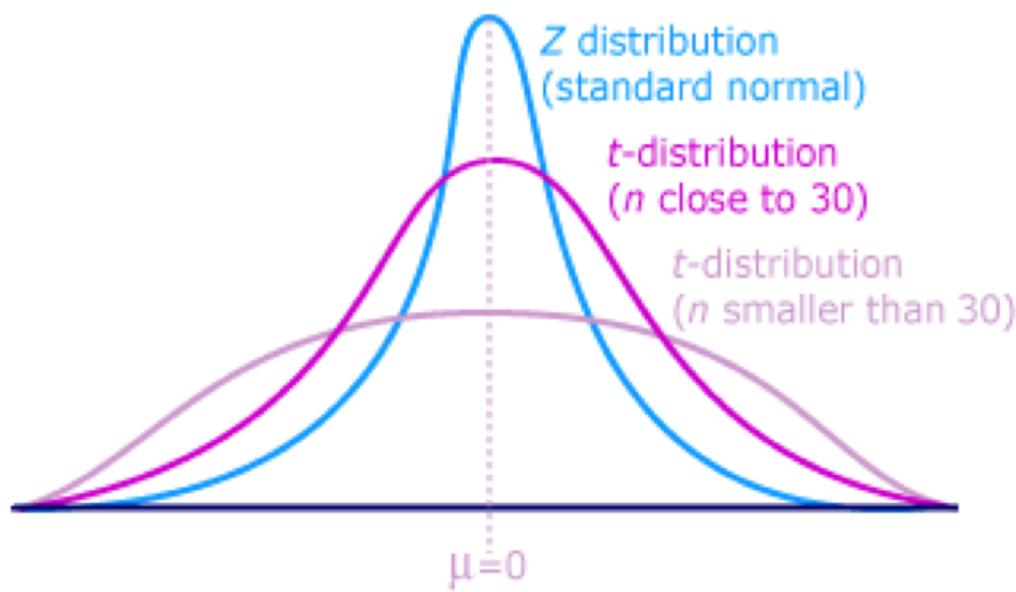
Confidence Interval for a Mean

- Recall that formula for a confidence interval is
  statistic $\pm$ *margin of error*.
- When we are making an inference about a population mean, the statistic will be our sample mean, $\bar{x}$.
- The critical value we use to find the margin of error for our calculation will be based on whether the population or sample standard deviation is known.
- When the population standard deviation is known, we use the formula $\bar{x} \pm z^*(\sigma/\sqrt{n})$
- When it is unknown, we will need to find the sample standard deviation, s, and use the formula $\bar{x} \pm t^*(s/\sqrt{n})$

  where t^* is the t-critical value based on $n - 1$ degrees of freedom.

- t^* is the critical value for a t-distribution.
- Here is a comparison of the t-distribution and the standard normal distribution:



- Finding the critical *t* value for confidence level CL and *df* degrees of freedom:

  $df = n - 1$

- Using R-Studio,
  - qt(CL +(1-CL)/2, *df*)
- With a TI-83/84 calculator,
  - invT(CL +(1-CL)/2, *df*)

- The assumptions for a population mean are:
  1. The sample must be an SRS from the population of interest.
  2. The data must come from a normally distributed population. If this is not the case or if we are unsure whether the population is normally distributed, the sampling distribution of must be normally distributed. $(n \geq 30)$

Example:

$\sigma = .005$

1. Suppose your class is investigating the weights of Snickers 1-ounce fun-size candy bars to see if customers are getting full value for their money. Assume that the weights are normally distributed with standard deviation = $\boxed{.005 \text{ ounces.}}$ Several candy bars are randomly selected and weighed with sensitive balances borrowed from the physics lab.

$n = 8$

The weights are: ⟨ 0.95, 1.02, 0.98, 0.97, 1.05, 1.01, 0.98, 1.00 ⟩

We want to determine a <u>90% confidence interval</u> for the true mean, .

a. What is the sample mean? $\bar{x} = .995$

b. Determine z^* and the margin of error.

$z^* = \text{InvNorm}(.95) = 1.645$

$SE = \sigma/\sqrt{n} = .005/\sqrt{8}$

ME $= .0029$

c. Determine the 90% confidence interval. (Show your work)

$.995 \pm .0029 = [.9921, .9979]$

d. Write a sentence that explains the significance of the confidence interval.

We are 90% confident that the true mean weight of the fun sized snickers bar is between .9921 ounces and .9979 ounces

Example:

$n = 16$ $\quad \bar{x} = 500 \quad s = 100 \quad t^* = \text{invT}(.95, 15) = 1.753$

2. A SRS of 16 seniors from HISD had a mean SAT-math score of 500 and a standard deviation of 100. We know that the population of SAT-math scores for seniors in the district is approximately normally distributed.

   a. Find the 90% confidence interval for the mean SAT-math score for the population of all seniors in the district.

   $$500 \pm 1.753\left(\frac{100}{\sqrt{16}}\right) = 500 \pm 43.825 = [456.175, 543.825]$$

   b. Explain the meaning of the above confidence interval.

   We are 90% confident the true mean SAT-math score for HISD seniors is between 456.175 and 543.825.

Example:

3. The effect of exercise on the amount of lactic acid in the blood was examined in an article for an exercise and sport magazine. Eight males were selected at random from those attending a week-long training camp. Blood lactate levels were measured before and after playing three games of racquetball, as shown in the accompanying table. Use this data to estimate the mean increase in blood lactate level using a 95% confidence interval.

df = 7

Player	1	2	3	4	5	6	7	8
Before	13	20	17	13	13	16	15	16
After	18	37	40	35	30	20	33	19
Increase = after-before	5	17	23	22	17	4	18	3

$\bar{x} = 13.625$ $s = 8.2797$ $t^* = \text{invT}(.975, 7) = 2.365$

$13.625 \pm 2.365\left(8.2797/\sqrt{8}\right)$

$[6.70, 20.545]$

Example:

4. A 95% confidence interval for the mean of a population is to be constructed and must be accurate to within 0.3 unit. A preliminary sample standard deviation is 2.9. Find the he smallest sample size n that provides the desired accuracy.

$ME = .3$

$z^* = 1.96 \qquad s = 2.9$

$.3 \geq 1.96 \left(\frac{2.9}{\sqrt{n}}\right)$

$\sqrt{n} \geq \frac{1.96(2.9)}{.3}$

$\sqrt{n} \geq 18.9467$

$n \geq 358.98$

$\boxed{n = 359}$