Confidence Interval for the Difference of Two Proportions

- The assumptions that need to be satisfied for a two-sample proportion are slightly different than those for a one-sample.
  1. Both samples must be independent SRSs from the populations of interest.
  2. The population sizes are both at least ten times the sizes of the samples.
  3. The number of successes and failures in both samples must all be ≥ 10.

- To make the comparison, we will need to find the difference of the two proportions, $\hat{p}_1 - \hat{p}_2$.
- The standard error for this difference is

$$\sqrt{\frac{\hat{p}_1\left(1-\hat{p}_1\right)}{n_1}+\frac{\hat{p}_2\left(1-\hat{p}_2\right)}{n_2}}$$

- So our formula for the confidence interval is:

$$\left(\hat{p}_1-\hat{p}_2\right)\pm z^*\sqrt{\frac{\hat{p}_1\left(1-\hat{p}_1\right)}{n_1}+\frac{\hat{p}_2\left(1-\hat{p}_2\right)}{n_2}}$$

Example:

The National Research Council of the Philippines reported that 210 of 361 members in biology are women, but only 34 of 86 members in mathematics are women. Establish a 96% confidence interval estimate of the difference in proportions of women in biology and mathematics in the Philippines. Interpret your results.

Biology = sample 1 $\hat{p}_1 = \frac{210}{361}$

Math = sample 2 $\hat{p}_2 = 34/86$

96% CI $z^* = \text{invNorm}\left(.96 + \frac{(1-.96)}{2}\right) = \text{invNorm}(.98)$

$z^* = 2.054$

$SE = \sqrt{\frac{210/361(1-210/361)}{361} + \frac{34/86(1-34/86)}{86}} = .0588$

$ME = (2.054)(.0588) = .1208$

$\left(\frac{210}{361} - \frac{34}{86}\right) \pm .1208 = [.0656, .3072]$