Section 5.4 - The Distribution of the Sample Mean

Suppose we take n measurements from a normal distribution with unknown mean. How can we approximate the mean?

Sample mean $= \sum x_i / n = \bar{x}$

Ex: Suppose the scores on Exam 1 are normally distributed and the average score is unknown. If 5 of the scores are

74	98	83	70	91

approximate the average score on the exam.

$$\bar{x} = \frac{74+98+83+70+91}{5} = 83.2$$

* distr. of sample means

Theorem:
If we sample from a normal distribution with mean μ and standard deviation σ, then the distribution of $\bar{X}$ is $N\left(\mu, \sigma^2/n\right)$.

$\ast$ = variance = σ^2/n ⇒ st. dev = $\sigma/\sqrt{n}$

Ex: Suppose the scores on Exam 1 are normally distributed with mean 74, and standard deviation 10. If 5 scores are taken at random, determine the probability that the average of those 5 scores is less than 70.

$\mu = 74 \quad \sigma = 10$

$n = 5$

$P(\bar{X} < 70) = \text{normalcdf}(-9999, 70, 74, 10/\sqrt{5})$

$= \text{pnorm}(70, 74, 10/\sqrt{5})$

$\mu_{\bar{X}} = 74$

$\sigma_{\bar{X}} = 10/\sqrt{5}$

$P\left(z < \frac{70-74}{10/\sqrt{5}}\right) = P(z < -.89) = \boxed{.1855}$

Ex: Suppose that the race times of a certain sprinter are normally distributed with mean 55 seconds and standard deviation 4 seconds. If a certain qualification round require the average of three sprints to be less than 54 seconds in order to advance, determine the probability that the sprinter advances.

$\mu = 55 \quad \sigma = 4 \qquad n = 3 \quad \bar{x} = 55 \quad \sigma_{\bar{x}} = 4/\sqrt{3}$

$$P(\bar{x} < 54) = \text{normalcdf}(-99999, 54, 55, 4/\sqrt{3})$$

$$= \text{pnorm}(54, 55, 4/\sqrt{3})$$

$$= .3325$$

The Central Limit Theorem

If $\bar{x}$ is the mean of a random sample $X_1, X_2, \ldots, X_n$ from a distribution with mean μ and finite variance σ^2, then the distribution of

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

approaches the standard normal distribution as n approaches infinity.

The larger the sample size ($n \geq 30$) the more "normal" the distribution of sample means becomes $\bar{X} \sim N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right)$