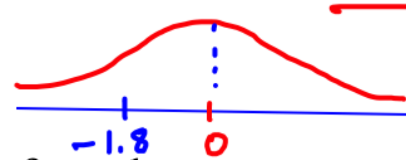
Review of Ch 4 Questions:

1. A student takes a standardized exam. The grader reports the student's standardized score (z-score) as -1.8. This indicates:

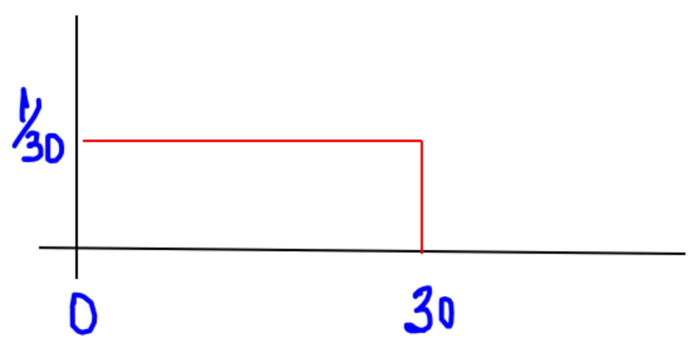


   a. The student scored lower than the average.
   b. The student scored less than one standard deviation from the average.
   c. A mistake has been made in calculating the score, since a standard score can never be negative.
   d. Both a and b, but not c.

2. Suppose you are going out for the evening with friends and they ask you to be ready to leave by 9:00pm. Your friends will arrive at a time T uniformly distributed between 9:00 and 9:30.

   a) State the distribution and its parameter(s):

$$f(x) = \begin{cases} 1/30 & 0 \le x \le 30 \\ 0 & \text{otherwise} \end{cases}$$



b) What is the probability that you will have to wait more than 20 minutes for your friends?

$$P(X>20) = 10\left(\tfrac{1}{30}\right) = \tfrac{1}{3} \qquad = 1 - P(X \le 20)$$

c) If at 9:20 your friends have not yet arrived, what is the probability that you have to wait at least 5 more minutes?

$$P(X \ge 25 \mid \underbrace{\text{already waited 20 min}}_{X \ge 20}) = \frac{5\left(\tfrac{1}{30}\right)}{\tfrac{1}{3}} = \frac{1}{2}$$

d) What is the probability that your friends will arrive at exactly 9:25?

$$P(X=25) = 0$$

e) What is the amount of time you would expect to wait for your friends? And what is the variance of the time waited?

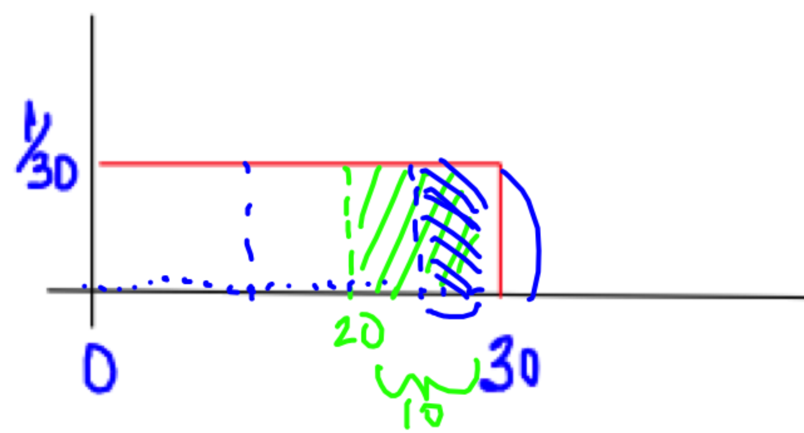
$$\sigma = \sqrt{75} \approx 8.66 \text{ min}$$

$$E[X] = \int_0^{30} x\, f(x)\,dx = \int_0^{30} \frac{x}{30}\,dx$$

$$= \frac{x^2}{60}\Big|_0^{30} = 15 \text{ min}$$

$$Var[X] = E[X^2] - (E[X])^2$$

$$= \int_0^{30} x^2 f(x)\,dx - (15)^2 = 75$$



3. A trucker drives between a fixed location in Los Angeles and Phoenix. The duration in hours of a round trip has an exponential distribution with parameter 1/20. Determine the probability of a round trip:

$\lambda = 1/20$

a) That takes at most 15 hours

$$P(X \le 15) = \int_0^{15} \frac{1}{20} e^{-x/20}\,dx = 1 - e^{-1/20(15)}$$

$$\approx .5276$$

b) That takes between 15 and 25 hours

$$P(15 \le X \le 25) = \int_{15}^{25} \frac{1}{20} e^{-x/20}\,dx$$

or $P(X \le 25) - P(X < 15)$

$$= .1859$$

c) That exceeds 25 hours

$$P(X>25) = 1 - P(X \le 25) = 1 - \int_0^{25} \frac{1}{20} e^{-x/20}\,dx$$

$$= 1 - .7135$$

$$= .2865$$

d) Find the mean and variance for the amount of time a trip will take

$$\mu = 1/\lambda = 20 \text{ hrs}$$

$$\sigma^2 = 1/\lambda^2 = 400$$

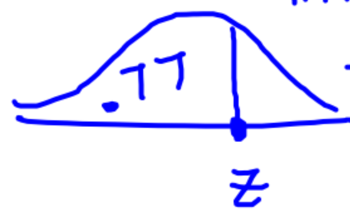
normal
$\mu = 1040 \quad \sigma = 192$

4. Suppose an applicant needs to score better than 77% of all GRE test takers to get accepted into this university.

a) What is the minimum score required to meet this criteria?

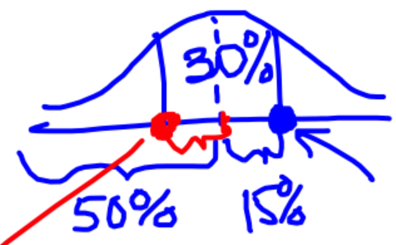
$$\frac{x - 1040}{192} = .738$$

$$x = 1181.8 \approx 1182$$



InvNorm(.77) $= \underline{\underline{.738}}$

b) What are the cutoff scores that would capture the middle 30% of applicants?



InvNorm(.65) = .385

InvNorm(.35) = −.385

$$\frac{x - 1040}{192} = -.385 \qquad \frac{x - 1040}{192} = .385$$

$$x = 966.08 \qquad x = 1113.92$$

(966, 1114)

5. The average stock price for companies making up the S&P 500 is \$30, and the standard deviation is \$8.20 (Business Week, Spring 2003). Assume the stock prices are normall distributed.

a) What is the probability a company will have a stock price of at least \$40?

$P(X \geq 40) = \text{normalcdf}(40, 100000, 30, 8.20)$

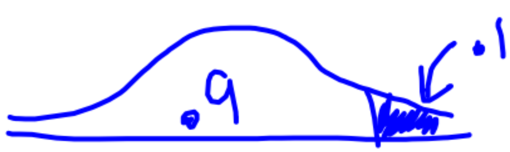
$1 - \text{pnorm}(40, 30, 8.2) \approx .1113$

b) What is the probability a company will have a stock price no higher than \$20?

$P(X \leq 20] = \text{normalcdf}(-100000, 20, 30, 8.2)$

$\text{pnorm}(20, 30, 8.2) \approx .1113$

c) How high does a stock price have to be to put a company in the top 10%?



$\text{invNorm}(.9) = \dfrac{X - 30}{8.2}$

$\text{InvNorm}(.9, 30, 8.2) \quad X = \$40.50$

6. A person with tuberculosis is given a chest x-ray. Four TB x-ray specialists examined each x-ray independently. If each specialist can detect TB 88% of the time when it is present, what is the probability that less than three specialists will detect the presence of TB?

binomial: $p = .88 \quad n = 4$

$P(X < 3) = P(X \le 2) = \text{binomialcdf}(4, .88, 2)$

$= .0732$

(pbinom(2,4,.88))

7. Let X be a random variable with the following distribution:

X	5	10	15	20	25
P(X)	.05	.30	.25	.25	.15
X^2	25	100	225	400	625

a. What is $P(X \leq 15)$? $.05 + .30 + .25 = .60$

$= V[X] = E[X^2] - (E[X])^2 = 33.1875 \quad \sigma = 5.76$

b. Find the expected value and standard deviation of X.

$$E[X] = 5(.05) + 10(.3) + 15(.25) + 20(.25) + 25(.15)$$
$$= 15.75$$

c. Find the expected value and standard deviation of Y, if Y = 4X – 3.

$$E[Y] = E[4X-3] = 4E[X] - 3 = 4(15.75) - 3 = 60$$

$$\sigma[Y] = 4\sigma_X = 4(5.76) \approx 23.04$$

8. A car wash loses \$30 on rainy days and gains \$120 on days when it does not rain. If the probability of rain is 0.15, what is the expected income for the car wash?

X	-30	120
$P(X)$	$.15$	$.85$

$$E[x] = -30(.15) + 120(.85) = \$97.50$$