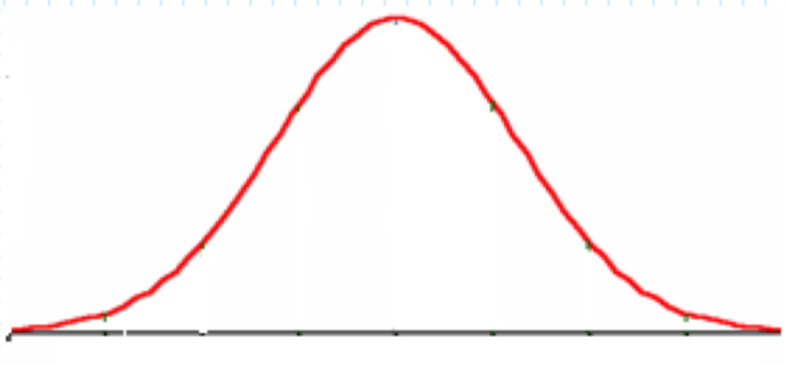
The Normal Distribution

- A density curve that is symmetric, single peaked and bell shaped is called a **normal distribution**.
- The normal distribution with mean μ and standard deviation σ is represented by **N(μ, σ)**.



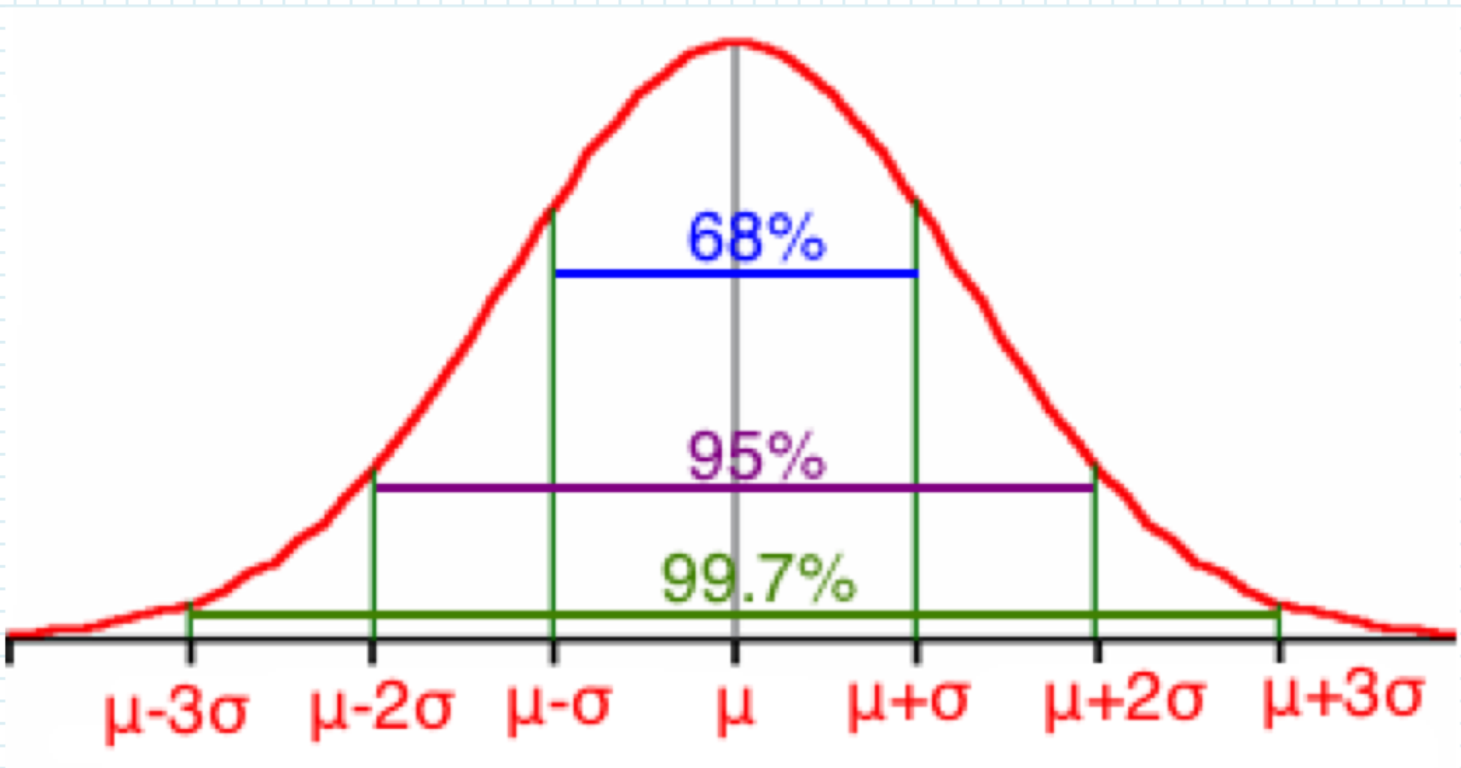
- **The Empirical Rule:**

  The Empirical Rule states if a distribution has a normal distribution,

  – Approximately 68% of all observations fall within one standard deviation of the mean.

  – Approximately 95% of all observations fall within two standard deviations of the mean.

  – Approximately 99.7% of all observations fall within three standard deviations of the mean.

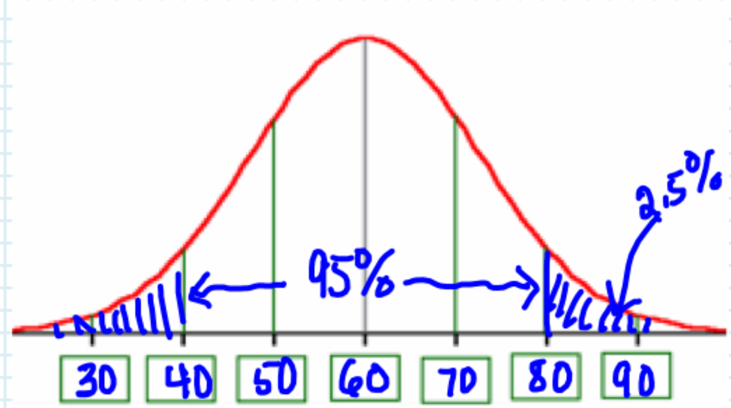

68%
95%
99.7%
μ-3σ
μ-2σ
μ-σ
μ
μ+σ
μ+2σ
μ+3σ

4.2 The Normal Distribution

Example:

1. The length of time needed to complete a certain test is normally distributed with mean 60 minutes and standard deviation 10 minutes.

   a. Sketch the distribution and shade in the area in question.



   b. What is the probability that someone will take between 40 and 80 minutes to complete the test?

   $60 \pm 2(10) = (40, 80)$

   95%

   c. Find the interval that contains the middle 68% of completion times for all people taking the test.

   $(50, 70)$

   d. What percent of people take more than 80 minutes to complete the test?

   $.05/2 = .025$ or 2.5%

- What if our values are not exactly within one, two or three standard deviations from the mean?
- Probabilities for these can still be found a number of ways, one of which we will explore in the next section.
- Using R, the probability $P(X < x)$ can be found with the command pnorm(X, μ, σ). Note that the command in R only gives the probability that X is less than a given value. If we need to find the probability that X is greater than the given value, we will need to subtract the answer from 1.
- With the TI-83 and TI-84 calculator, the command is normalcdf(lower_limit, upper_limit, μ, σ).

Example:

2. Refer to example 1. $N(60, 10)$

   a. What is the probability that someone will take less than 45 minutes to complete the test?

   $P(x < 45) = .0668$

   b. What is the probability that someone will take more than 30 minutes to complete the test?

   $P(x > 30) = .9987$

   c. What is the probability that someone will take between 30 and 45 minutes to complete the test?

   $P(30 < x < 45) = .065457$

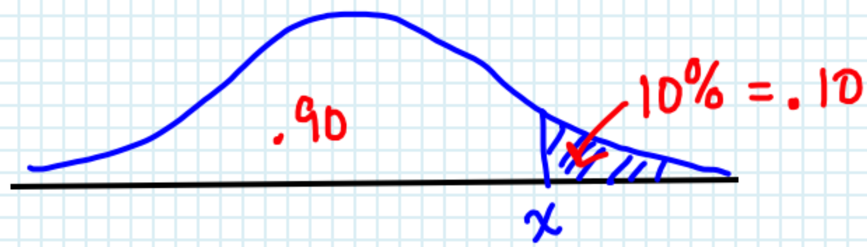
- If we know the probability and want to find the value of *x* such that P(*X* < *x*), we use one of the following commands:
  - Using the TI-83/84 calculator: invNorm(probability, mean, standard dev)
  - Using R-Studio:

    qnorm(probability, mean, standard dev)

4.2 The Normal Distribution

Example:

3. Using the data from example 1, how long would it take someone to finish a test if they are in the top 10% of the times?



$P(X < x) = .90$

$\text{invNorm}(.90, 60, 10) = 72.82$