Binomial Distributions

- A **Bernoulli Trial** is a random experiment with the following features:
  - The outcome can be classified as either a success or a failure (only two options and each is mutually exclusive).
  - The probability of success is p and probability of failure is $q = 1 - p$.
- A **Bernoulli random variable** is a variable assigned to represent the successes in a Bernoulli trial.
- If we wish to keep track of the <u>number</u> of successes that occur in repeated Bernoulli trials, we use a **binomial random variable.**

- A **binomial experiment** occurs when the following conditions are met:
  - Each trial can result in one of only two mutually exclusive outcomes (success or failure).
  - There are a fixed number of trials.
  - Outcomes of different trials are independent.
  - The probability that a trial results in success is the same for all trials.

- The random variable X = number of successes of a binomial experiment is a **binomial distribution** with parameters p and n where p represents the probability of a success and n is the number of trials.
- The possible values of X are whole numbers that range from 0 to n. As an abbreviation, we say X~B(n, p).
- **Binomial probabilities** are calculated with the following formula:

$$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}={}_nC_k p^k(1-p)^{n-k}$$

- In R-Studio,
  - $P(X = k)$ = dbinom(k,n, p)
  - $P(X \leq k)$ = pbinom(k,n, p)
  - $P(X > k) = 1$ - binom(k,n, p)

- With a TI-83/84 calculator,
  - $P(X = k)$ = binompdf(n, p, k)
  - $P(X \leq k)$ = binomcdf(n, p, k)
  - $P(X > k) = 1$ - binomcdf(n, p, k)

- The mean and variance of a binomial distribution are computed using the following formulas:

$$\mu_X = E\left[X\right] = np$$

$$\sigma_X^2 = np\left(1 - p\right)$$

Example:

1. A fair coin is flipped 30 times. $n = 30 \quad p = \frac{1}{2}$

a. What is the probability that the coin comes up heads exactly 12 times?

$$P(X=12) = {}_{30}C_{12} \left(\tfrac{1}{2}\right)^{12} \left(1-\tfrac{1}{2}\right)^{30-12} = .0806$$

b. What is the probability the coin comes up heads less than 12 times?

$$P(X<12) = P(X \le 11) = .1002$$

c. What is the probability the coin comes up heads more than 12 times?

$$P(X>12) = 1 - P(X \le 12) = 1 - \text{binomcdf}(30, .5, 12) = .8192$$

d. What is the expected number of heads?

$$E[X] = np = 30\left(\tfrac{1}{2}\right) = 15$$

Example:

$p = .8 \quad n = 5$

2. Suppose it is known that 80% of the people exposed to the flu virus will contract the flu. Out of a family of five exposed to the virus, what is the probability that:

a. No one will contract the flu? $P(X=0) = {}_5C_0 (.8)^0 (1-.8)^{5-0}$

$= \text{binompdf}(5, .8, 0) = .00032$

b. All will contract the flu?

$P(X=5) = {}_5C_5 (.8)^5 (1-.8)^{5-5} = .3277$

c. Exactly two will get the flu?

$P(X=2) = {}_5C_2 (.8)^2 (1-.8)^{5-2} = .0512$

d. At least two will get the flu?

$P(X \geq 2) = 1 - P(X \leq 1) = .9933$

0 1 | 2 3 4 5

Example:

3. Using the binomial distribution in example 2,

a. Let X = number of family members contracting the flu. Create the probability distribution table of X.

X	0	1	2	3	4	5
P(x)	.00032	.0064	.0512	.2048	.4096	.32768

b. Find the mean and variance of this distribution.

$$E[X] = 0(.00032) + 1(.0064) + 2(.0512) + 3(.2048) + 4(.4096) + 5(.32768)$$

$$= np = 5(.8) = 4$$

$$\sigma_x^2 = np(1-p) = 5(.8)(.2) = .8$$