Section 10.2 – Tukey's Method (The *T* Method)

The T-method is used to determine *which* pair (or pairs) of means differ significantly.

Tukey's Procedure (the *T* Method)

The *T* Method for Identifying Significantly Different μ_i's

Select α, extract $\boxed{Q_{\alpha,I,I(J-1)}}$ from Appendix Table A.10, and calculate $w = Q_{\alpha,I,I(J-1)} \cdot \sqrt{\text{MSE}/J}$. Then list the sample means in increasing order and underline those pairs that differ by less than w. Any pair of sample means not underscored by the same line corresponds to a pair of population or treatment means that are judged significantly different.

$$Q_{\alpha,I,I(J-1)} = \text{qtukey}(1-\alpha, I, I(J-1))$$

- $$W = Q_{\alpha,I,I(J-1)} \sqrt{\frac{MSE}{J}}$$
- distance allowed between μ_i's

An experiment was carried out to compare five different brands of automobile oil filters with respect to their ability to capture foreign material. Let μ_i denote the true average amount of material captured by brand i filters ($i = 1, \ldots, 5$) under controlled conditions. A sample of nine filters of each brand was used, resulting in the following sample mean amounts: $\bar{x}_{1\cdot} = 14.5$, $\bar{x}_{2\cdot} = 13.8$, $\bar{x}_{3\cdot} = 13.3$, $\bar{x}_{4\cdot} = 14.3$, and $\bar{x}_{5\cdot} = 13.1$. Table 10.3 is the ANOVA table summarizing the first part of the analysis.

Table 10.3 ANOVA Table for Example 10.5

Source of Variation	df	Sum of Squares	Mean Square	f
Treatments (brands)	4	13.32	3.33	37.84
Error	40	3.53	$\boxed{.088}$	
Total	44	16.85		

MSE

$I-1=4 \Rightarrow I=5.$

$I(J-1) = 5(J-1) = 40$

$J-1 = 8$

$J = 9$

$Q_{.05,5,40} = \text{qtukey}(.95, 5, 40) = 4.039$

$w = 4.039\sqrt{\frac{.088}{9}} \approx .4$

$\bar{x}_5$	$\bar{x}_3$	$\bar{x}_2$	$\bar{x}_4$	$\bar{x}_1$
13.1	13.3	13.8	14.3	14.5

(3,2) (3,4) (3,1)

(5,2) (5,4) (5,1)

(2,4) (2,1)

A biologist wished to study the effects of ethanol on sleep time. A sample of 20 rats, matched for age and other characteristics, was selected, and each rat was given an oral injection having a particular concentration of ethanol per body weight. The rapid eye movement (REM) sleep time for each rat was then recorded for a 24-hour period, with the following results:

Treatment (concentration of ethanol)						$\Sigma x_{i\cdot}$	$\bar{x}_{i\cdot}$
0 (control)	88.6	73.2	91.4	68.0	75.2	396.4	79.28 ①
1 g/kg	63.0	53.9	69.2	50.1	71.5	307.7	61.54 ②
2 g/kg	44.9	59.5	40.2	56.3	38.7	239.6	47.92 ③
4 g/kg	31.0	39.6	45.3	25.2	22.7	163.8	32.76 ④

$x_{\cdot\cdot} = 1107.5 \quad \bar{x}_{\cdot\cdot} = 55.375$

$I = 4, \ J = 5$

Does the data indicate that the true average REM sleep time depends on the concentration of ethanol? (This example is based on an experiment reported in "Relationship of Ethanol Blood Level to REM and Non-REM Sleep Time and Distribution in the Rat," *Life Sciences*, 1978: 839–846.)

Table 10.4 SAS ANOVA Table

Analysis of Variance Procedure

Dependent Variable: TIME

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	5882.35750	1960.78583	21.09	0.0001
Error	16	1487.40000	92.96250		
Corrected Total	19	7369.75750			

pairs that differ significantly:

(4, 2) (4, 1)

(3, 1)

(2, 1)

$Q_{.05, 4, 16} = 4.05 \qquad W = 4.05\sqrt{\dfrac{92.96250}{5}} = 17.5$