Introduction to Statistics
Chi-Square Test for Homogeneity

- The Chi-Square Test for Homogeneity is a test used to determine whether frequency counts are distributed identically across different populations. (In other words, whether equal proportions exist in a series of populations).
- The null and alternate hypothesis are:
  - H_0: The distributions are the same
  - H_a: The distributions are not the same

- The conditions to use χ^2 test for homogeneity are:
  1. Each sample is an SRS from its respective population.
  2. Each population is at least 10 times as large as its respective sample.
  3. The variable we are interested in is categorical.
  4. The expected frequency count for each value is at least 5.

- We conduct this test much like the Chi-square test of independence.
- The formula to find the expected count is:

$$\text{expected count} = \frac{\text{row total} \times \text{column total}}{n}$$

- Where n is the grand total of all values.
- The test statistic is: $\chi^2 = \sum_{allcells} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$
- The p-value: $P(\chi^2_k > \chi^{2)}$, where χ^2_k represents a Chi-square distribution with $df = (r - 1)(c - 1)$ degrees of freedom.

Example:

H_0: distributions for tv shows same for men & women

H_a: distributions are different

Suppose a study was conducted to determine the t.v. viewing habits of men and women. A random sample of 250 adults (100 men and 150 women) was taken and their results are:

$df = (2-1)(3-1) = 2$

Obs.	The Blacklist	American Idol	NCIS	Total
Men	40	15	45	100
Women	50	62	38	150
Total (col)	90	77	83	250

row totals

$n = 250$

$\alpha = .05$

Test at the 5% significance level to determine if the men's t.v. preferences are different from the women's.

Exp

men	$\frac{100 \times 90}{250} = 36$	$\frac{100 \times 77}{25} = 30.8$	$\frac{83 \times 100}{250} = 33.2$
Women	$\frac{150 \times 90}{250} = 54$	$\frac{150 \times 77}{250} = 46.2$	$\frac{150 \times 83}{250} = 49.8$

$\chi^2 = 25.47$

$\chi^2 = 24.57$

pvalue: $p(\chi^2 > 24.57) = 4.614 \times 10^{-6} < \alpha$

Reject