Problem 1. Given the points $A(1, -2, 1), B(3, 2, 2), C(-2, 1, -5)$.
(a) Find the norm of the vector that goes from point $A$ to point $C$.
(b) Find an equation for the plane that contains $A$, $B$, and $C$.
(c) Find an equation for the sphere that has points $A$ and $B$ as the endpoints of its diameter.

Problem 2. Given the vectors $a = 2i - k, b = 2i + 3j - k, c = -i + 3j - 2k$
(a) Find a unit vector in the direction of vector $b$.
(b) Find: $\text{comp}_b a$
(c) Find: $\text{proj}_b c$
(d) Determine the measure of the angle between $a$ and $b$.

Problem 3. Given the plane $P: 3x - y + 4z = 3$, the line $L: \frac{x-1}{-2} = \frac{y+4}{2} = \frac{z+3}{2}$, and the point $A(-3, 0, 5)$:
(a) Determine whether $P$ and $L$ are parallel.
(b) Determine the parametric equations for the line $R$ that passes through $A$ and is parallel to $L$.
(c) Determine the parametric equations for the line $M$ that passes through $A$ and is perpendicular to $P$.

Problem 4. Given the planes $P_1: x + 4y - z = 10$, $P_2: 3x - y + 2z = 4$, and the point $A(-2, 1, 0)$:
(a) Determine the equation for the plane that contains $A$ and is parallel to $P_1$.
(b) Determine whether $P_1$ and $P_2$ are parallel. If not, find symmetric equations for the line of intersection of $P_1$ and $P_2$.
(c) Find the cosine of the angle between $P_1$ and $P_2$.

Problem 5. Let $f(t) = (2t + 1)i - \cos (\pi t)j$ and $g(t) = (t - 1)i + \cos (\pi t)j$
(a) Find $\lim_{t \to 3} f(t)$.
(b) Calculate $[f(t) \cdot g(t)]'$.

Problem 6. The position of an object at time $t$ is given by the vector function: $r(t) = (e^{-t} \sin t)i + (e^{-t} \cos t)j + 3tk$
Determine:
(a) The velocity vector: $v(t)$.
(b) The speed of the object at time $t$.
(c) The acceleration vector: $a(t)$.
Problem 7.  At time $t$ a particle moves so that:

$$
\mathbf{r}(t) = \left(\frac{1}{2}t^2 + 1\right) \mathbf{i} + \left(\frac{1}{3}t^3 - 1\right) \mathbf{j}
$$

(a) Find the position at time $t = 4$.
(b) Find the distance travelled from $t = 0$ to $t = 3$.

Problem 8.  The vector function: $\mathbf{r}(t) = 2t \mathbf{i} + t^2 \mathbf{j} + \frac{1}{3}t^3 \mathbf{k}$ determines a curve $C$ in space.

(a) Find scalar parametric equations for the tangent line to $C$ at the point where $t = 3$.
(b) Find the length of $C$ from $t = 0$ to $t = 3$.
(c) Determine the unit tangent vector $\mathbf{T}$ and the principal normal vector $\mathbf{N}$ at $t = 1$.

Problem 9.  The vector function $\mathbf{r}(t) = (\cos t + t \sin t) \mathbf{i} + (\sin t - t \cos t) \mathbf{j} + \frac{1}{2}\sqrt{3}t^2 \mathbf{k}$ determines a curve $C$ in space.

(a) Find the unit tangent vector $\mathbf{T}(t)$ and the principal normal vector $\mathbf{N}$.
(b) Find the curvature, $\kappa$ of $C$.
(c) Determine the tangential and normal components of acceleration; express the acceleration vector, $\mathbf{a}(t)$ in terms of $\mathbf{T}$ and $\mathbf{N}$.

Problem 10.  Find the domain of the function:

$$
f(x, y) = \frac{2 \cos (x + y)}{\sqrt{9 - x^2 - y^2}}
$$

Problem 11.  Identify the level curves of the given surface.

(a) $f(x, y) = e^{2x^2 + 3y^2}$
(b) $f(x, y) = \ln(x - 4y^2)$

Problem 12.  Given $f(x, y) = \frac{2y}{x^2 + y}$, find $\lim_{(x,y) \to (0,0)} f(x, y)$ along the following paths:

(a) the $x$-axis
(b) the $y$-axis
(c) the parabola $y = 3x^2$
(d) what can you conclude about $\lim_{(x,y) \to (0,0)} f(x, y)$? Why?

Problem 13.  Let $f(x, y) = y^2 e^{xy} + \frac{x}{y}$. Calculate all first and second partial derivatives.
Problem 14. Let \( f(x,y) = x^2 ye^{x-1} + 2xy^2 \) and \( F(x, y, z) = x^2 + 3yz + 4xy. \)

(a) (i) Find the gradient of \( f. \)

(ii) Determine the direction in which \( f \) decreases most rapidly at the point \((1, -1).\) At what rate is \( f \) decreasing?

(b) (i) Find the gradient of \( F. \)

(ii) Find the directional derivative of \( F \) at the point \((1, 1, -5)\) in the direction of the vector 
\( \mathbf{a} = 2 \mathbf{i} + 3 \mathbf{j} - \sqrt{3} \mathbf{k}. \)