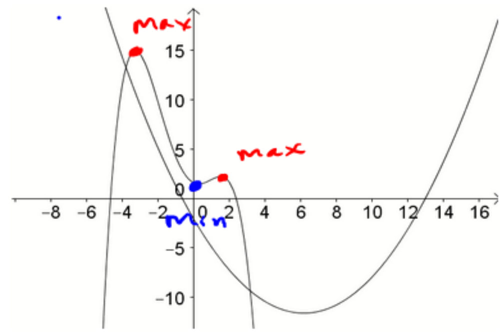
Midterm 1 information

Test is Friday, March 1 - Saturday, March 2 at CASA.
Test is given by appointment only. Make a reservation at casa.uh.edu on the ~~online assignments~~ tab. Proctored Exams
Test is 18 questions, 12 multiple choice and 6 free response. 48 points from MC; 52 points from FR
Turn in your Written Review (if you did it) when you check in at CASA. This review is optional. It's not the in-class review....it's on the course homepage, middle column near the bottom. Up to 5 points of Extra Credit on the Test.
Also take the practice test. It is also worth up to 5 points of Extra Credit on the Test.
Remote testers - if I haven't heard from you - NOW IS THE TIME!!! If you are testing remotely, you can send your Written Review by email.

Math 1314
Review for Test 1

Suppose $f(x) = -0.15x^4 - 0.28x^3 + 1.65x^2 - 0.93x + 1.6$ and $g(x) = 0.25x^2 - 3.1x - 2$.



1. Graph *f* and find an appropriate viewing window.
2. Find any zeros of *f*.

A = (-4.6354, 0)
B = (2.404, 0)

Root [polynomial]

3. Find any local (relative) maxima for *f*.

C = (-3.2432, 14.9278) max
D = (0.3122, 1.4605) min
E = (1.5311, 2.2148) max

4. Find any local (relative) minima for *f*.

5. Graph *g* on the same coordinate plane and adjust the viewing window if needed.
6. Find any zeros of *g*.

Root [polynomial]

7. Find any local extrema for *g*.

Extremum [polynomial]

8. Find any points where *f* and *g* intersect.

Intersect [object, object]

F = (-3.7716, 13.2484)
G = (3.1986, -9.3579)

Suppose $h(x)=\dfrac{x^2\ln\left(x^2-1\right)}{(x-2)^2}$.

9. Set up an table with start value 3 and increment 0.75.

A	B
3	18.715
3.75	11.7999
4.5	9.5823
5.25	8.5577
6	7.9995
6.75	7.6674
7.5	7.4601
8.25	7.3279
9	7.2438
9.75	7.1918
10.5	7.1623

10. Find $h(3.1)$ and ~~$h(\ldots)$~~.

$h(3.1) = 17.0988$

Suppose $f(x)=\begin{cases}2x-1, & x\le -1\\ x^2+3, & x>-1\end{cases}$

~~11. Set up the table with a start value of 0 and an increment of 1.~~

12. Find $\overset{\text{top}}{f(-16)}$ and $\overset{\text{bottom}}{f(34)}$.

$f(-16) = 2(-16)-1 = -32-1 = -33$

$f(34) = 34^2 + 3 = 1159$

13. Find $\overset{\text{top}}{f(-1)}$. $= 2(-1) - 1 = -2-1 = -3$

14. Determine $\lim\limits_{x\to -1^-} f(x)$, $\lim\limits_{x\to -1^+} f(x)$, and $\lim\limits_{x\to -1} f(x)$ (if it exists).

$\lim\limits_{x\to -1^-} f(x) = 2(-1) - 1 = -3$

$\lim\limits_{x\to -1^+} f(x) = (-1)^2 + 3 = 4$

$\lim\limits_{x\to -1} f(x)$ dne

$-3 \neq 4$

15. Is the function continuous at x = -1?

$x = -1$ it does $f(-1) = -3$

does $\lim\limits_{x\to -1} f(x)$ exist? No

f is not continuous at $x = -1$ b/c $\lim\limits_{x\to -1} f(x)$ dne

Suppose you are given this set of data:

X	2	4	5	7	8	11	14	18
y	23	25	27	30	32	38	40	51

16. Enter the data into the lists on your calculator.
17. Adjust the viewing window so that all points are included.
18. Find a linear regression model and the value for r^2. Use the model to predict the value of y when x = 25.

(a) $f(x) = 1.7106\,x + 18.496$

(b) $r^2 = 0.982$

(c) $f(25) = 61.2612$

19. Find a cubic regression model and the value for R^2. Use the model to predict the value of y when x = 25.

(a) $g(x) = 0.0034\,x^3 - 0.0751\,x^2 + 2.0116\,x + 18.6848$

(b) $R^2 = 0.9892$

(c) $g(25) = 75.8388$

20. Find an exponential regression model and the value for R^2. Use the model to predict the value of y when x = 25.

(a) $h(x) = 21.0671\,e^{0.0492x}$

(b) $R^2 = 0.9865$

(c) $h(25) = 72.0473$

21. Find a power regression model and the value for R^2. Use the model to predict the value of y when x = 25.

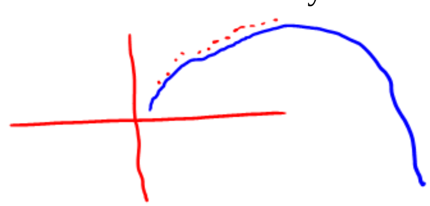
(a) $p(x) = 16.0249\,x^{0.3569}$

(b) $R^2 = 0.9097$

(c) $p(25) = 50.5479$

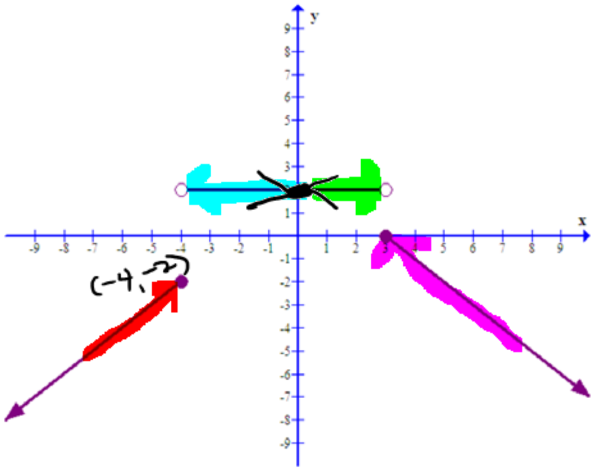
22. Look at the graphs of the four models. Which ones seem to fit the data and the trend? Which ones don't? Which one would you select to work with? Why is the value of R^2 alone not enough to make a choice?

$R^2 = .9998$



Find each limit:

For problems 23 – 29, use the graph.



23. $\lim_{x \to -4^-} f(x)$ = -2

24. $\lim_{x \to -4^+} f(x)$ = 2

25. $\lim_{x \to -4} f(x)$ dne b/c $\lim_{x \to -4^-} f(x) \neq \lim_{x \to -4^+} f(x)$

26. $\lim_{x \to 3^-} f(x)$ = 2

27. $\lim_{x \to 3^+} f(x)$ = 0

28. $\lim_{x \to 3} f(x)$ dne

29. $\lim_{x \to 0} f(x)$ = 2

indeterminate!!

30. $\lim_{x \to 0} \frac{x^2 - 2x}{x}$ $= -2$

$\frac{0^2 - 2(0)}{0} = \frac{0}{0}$

Limit[function, value]

31. $\lim_{x \to 2} \frac{x+7}{x-2}$ dne

$\frac{2+7}{2-2} = \frac{9}{0}$

32. $\lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x}$ $= 0.25$

$\frac{\sqrt{0+4} - 2}{0} = \frac{0}{0}$

indeterminate!!

Limit [f, 0] = 0.25

33. $\lim_{x \to 2} \frac{x^2 \sqrt{x^2 + 3}}{4x + 1}$ $= \frac{2^2 \sqrt{2^2 + 3}}{4(2)+1} = \frac{4\sqrt{4+3}}{8+1} = \boxed{\frac{4\sqrt{7}}{9}}$

34. $\lim_{x \to \infty} \frac{2x^2 - 8x}{3 - 4x^2}$ $=$

deg(num) = 2

deg(den) = 2

$\frac{2}{-4} = \boxed{-\frac{1}{2}}$

BOBO

~~BOTNX~~

EATS DC

35. $\lim_{x \to \infty} \frac{x^3 + 5x^2 - 7x - 1}{2 + x - 7x^2}$ $= \lim_{x \to \infty} \frac{\frac{\cancel{x^3}}{\cancel{x^2}} + \frac{5\cancel{x^2}}{\cancel{x^2}} - \frac{7\cancel{x}}{x^{\cancel{2}}} - \frac{1}{x^2}}{\frac{2}{x^2} + \frac{\cancel{x}}{x^{\cancel{2}}} - \frac{7\cancel{x^2}}{\cancel{x^2}}}$

$\lim_{x \to \infty} \frac{x + 5 - \frac{7}{x} - \frac{1}{x^2}}{\frac{2}{x^2} + \frac{1}{x} - 7}$ $\frac{7}{x} \to 0$, $\frac{1}{x^2} \to 0$ $= \lim_{x \to \infty} \left(\frac{x+5}{-7}\right) = \boxed{-\infty}$

0, 0

36. $\lim_{x \to -\infty} \frac{x^3 + 5x^2 - 7x - 1}{2 + x^2 - 7x^4}$

deg(num) = 3

deg(den) = 4

$\boxed{0}$

Find each derivative:

Step 4 $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} =$

37. Use $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to find the derivative of $f(x)=3x^2+4x-5$.

$\lim_{h\to 0} (6x+3h+4) = \boxed{6x+4}$

Step 1
$f(x+h) = 3(x+h)^2 + 4(x+h) - 5$
$= 3(x+h)(x+h) + 4x + 4h - 5$
$= 3(x^2 + xh + xh + h^2) + 4x + 4h - 5 = 3x^2 + 6xh + 3h^2 + 4x + 4h - 5$

Step 2
$f(x+h) - f(x) = 3x^2 + 6xh + 3h^2 + 4x + 4h - 5 - (3x^2 + 4x - 5)$
$= \cancel{3x^2} + 6xh + 3h^2 + \cancel{4x} + 4h \cancel{-5} - \cancel{3x^2} - \cancel{4x} \cancel{+5}$
$= 6xh + 3h^2 + 4h = h(6x + 3h + 4)$

Step 3
$\frac{f(x+h)-f(x)}{h} = \frac{\cancel{h}(6x+3h+4)}{\cancel{h}} = 6x+3h+4$

$\boxed{f'(x) = 6x+4}$

38. Use $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to find the derivative of $f(x)=-2x^2-8x-5$ when x = 1.

$f(x+h) = -2(x+h)^2 - 8(x+h) - 5 = -2(x+h)(x+h) - 8x - 8h - 5$
$= -2(x^2 + xh + xh + h^2) - 8x - 8h - 5 = -2x^2 - 4xh - 2h^2 - 8x - 8h - 5$

$f(x+h) - f(x) = -2x^2 - 4xh - 2h^2 - 8x - 8h - (-2x^2 - 8x - 5)$
$= \cancel{-2x^2} - 4xh - 2h^2 \cancel{-8x} - 8h \cancel{-5} + \cancel{2x^2} + \cancel{8x} \cancel{+5} = -4xh - 2h^2 - 8h$

$\frac{f(x+h)-f(x)}{h} = \frac{\cancel{h}(-4x-2h-8)}{\cancel{h}} = -4x-2h-8$

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} (-4x-2h-8) = -4x-8$

$\boxed{f'(x) = -4x-8}$

39. Find the derivative: $f(x)=5x^4-3x^3+8x^2+7x-1$

$\boxed{f'(x) = 20x^3 - 9x^2 + 16x + 7}$

40. Find $f'(+2)$ if $f(x)=\dfrac{-e^{x}\sqrt{x^{2}+5}}{(x+2)^{2}}$. Find $f''(+2)$.

$f'(2) = -1.0006$

$f''(2) = -0.9129$

41. Find $f'(2000)$ if $f(x)=-0.005x^{4}+8.17x^{3}-0.014x^{2}+1.76x+12.3875$. Find $f''(2000)$.

$f'(2000) = -61960054.24$

$f''(2000) = -141960.028$

42. Find the average rate of change of $f(x)=0.28x^{2}-11.15x+12.4$ on the interval [-1.8, 3.7].

a b

$\dfrac{f(b)-f(a)}{b-a}$

$f(3.7)$

$f(-1.8)$

$3.7-(-1.8)$

$\dfrac{f(3.7)-f(-1.8)}{3.7-(-1.8)} = \boxed{-10.618}$

Continuity

Determine if the function is continuous at x = 1.

43. $f(x)=\begin{cases} x^{2}-3x+2, & x>1 \\ x^{2}-2x+1, & x\le 1 \end{cases}$

3 part test

① $f(1) = 1^2-2(1)+1=0$

$f(1)=0$

② $\lim_{x\to 1} f(x) = 0$

$\lim_{x\to 1^-} f(x) = 1^2-2(1)+1=0$

$\lim_{x\to 1^+} f(x) = 1^2-3(1)+2=0$

③ are they the same

$f(1) = \lim_{x\to 1} f(x) = 0$

$\boxed{\text{yes, } f \text{ is continuous @ } x=1}$

Rates of Change

44. Find the slope of the line that is tangent to the graph of $f(x)=3x^3-8x+7$. Write an equation of the tangent line at the point (1, 2).

$f'(x) = 9x^2 - 8$

slope of tan. line: $f'(1) = 9 \cdot 1^2 - 8 = 9 - 8 = 1$

equation of tan line:

$y = mx + b$

$2 = 1 \cdot 1 + b$

$2 = 1 + b$

$1 = b$

$y = 1x + 1$

$\boxed{y = x + 1}$

45. A ball is thrown upward with initial velocity 136 feet per second from the top of a building that is 84 feet high. (1) Write a function that gives the height of the ball at time *t*. Then find the velocity three seconds after the ball is thrown.

$h(t) = -16t^2 + 136t + 84$

$= -16t^2 + v_0 t + h_0$

$v(t) = h'(t) = -32t + 136$

$v(3) = h'(3) = -32(3) + 136 = -96 + 136$

$= 40$

$\boxed{40 \text{ ft/sec}}$

46. A ball is thrown upward with initial velocity 29 meters per second from the top of a building that is 51 meters high. (1) Write a function that gives the height of the ball at time *t*. (2) Then find the velocity three seconds after the ball is thrown.

$h(t) = -4.9t^2 + v_0 t + h_0$

(1) $h(t) = -4.9t^2 + 29t + 51$

(2) $v(t) = h'(t) = -9.8t + 29$

$v(3) = h'(3) = -9.8(3) + 29$

$= -0.4$

$\boxed{\text{Velocity is } -0.4 \text{ m/sec}}$

47. The population of a city is given by the function $P(t) = \frac{-1}{3}t^3 + 2.36t^2 + 0.891t + 1.63$ where $P(t)$ is given in millions and t is given in number of years since the beginning of 2005.

FV
ROC
AROC

A. What was the population of the city at the beginning of 2008? FV

$P(3) = 16.543$

$\boxed{\text{16.543 million people}}$

B. At what rate was the population of the city changing at the beginning of 2010?

ROC $P'(5) = -0.509$

$\boxed{\text{decreasing at therate of 0.509 million people per year}}$

48. A company finds that the fixed monthly expenses for producing x of the product are \$174,976 and the variable costs are \$15.00 per item. The company sells the product for \$47.00.

A. Write cost, revenue and demand functions using this information.

$C(x) = 15x + 174976$

$R(x) = 47x$

$P(x) = 47x - (15x + 174976)$

$= 47x - 15x - 174976 = \boxed{32x - 174976}$

B. Find the break even point.

$C = R$

$15x + 174976 = 47x$
$-15x \qquad -15x$

$\frac{174976}{32} = \frac{32x}{32}$

$5468 = x$

$R(5468) = 256996$

$\boxed{(5468, 256996)}$

49. The demand for a company's product can be modeled by the function $d(x) = 210 - 0.03x$ and the manufacturer will supply the product to the marketplace according to the model $s(x) = 0.0333x + 100$, where x is the number of units demanded and $d(x)$ is the unit price. Find the equilibrium quantity and price.

$d = s$

$210 - 0.03x = 0.0333x + 100$

$110 - 0.03x = 0.033x$

$+0.03x \qquad +0.03x$

$\dfrac{110}{0.063} = \dfrac{0.063x}{0.063}$

$1746.03 \approx x$

$210 - 0.03(1746) = 157.62$

equil. quantity
1746 units

equil. price
\$157.62