Math 1314
Lesson 2

Continuing with the introduction to GGB

Example 10: The path of a small rocket is modeled by the function $h(t) = -16t^2 + 128t + 12$ where initial velocity is 128 feet per section and initial height is 12 feet. The model gives the height of the rocket in feet, t seconds after launch. Find the height of the rocket 2 seconds after launch, 4 seconds after launch, 5 seconds after launch and 8 seconds after launch.

POPPER 1: Suppose $h(t) = -16t^2 + 128t + 12$. Find $h(3)$.

252   B. 540   C. 348   D. 249

Graphing

Example 11: Graph the function $f(x) = -2x - 3$.

Step 1: Enter the function in the input line.

Step 2: Resize the graphics window, if needed, to get a good view of the function.

Methods for resizing

How do you return to a standard viewing window?

Example 12: Graph the function $g(x) = x^3 - 2x^2 - 9x + 18$ and resize the graphics window, if needed, to find a good view of the function.

How do you resize the graphics window?

Example 13: Graph the function $h(x) = -3x^3 + 3x^2 + 96x - 180$ and find an appropriate viewing window.

Most of the work we will do in GGB will require the use of commands that are built into the program.

Where can you access a list of commands?

Finding Some Features of a Graphed Function

You can find the zeros (also called roots or *x* intercepts) of a function using GGB.

Example 14: Suppose $g(x) = x^3 - 2x^2 - 9x + 18$. Find the zeros of the polynomial function.

How do we know this is a polynomial function?

Step 1: Enter the function in the input line. Graph and resize if needed.

Step 2: Find the zeros of the polynomial function.

What's the command?

Example 15: Suppose $f(x) = \dfrac{x^2 - 3x - 5}{x^2 - 3}$. Find the zeros of the function.

Is this a polynomial function? How can we tell?

Step 1: Enter the function in the input line. Graph and resize if needed.

Step 2: Find the zeros of the function.

What's the command?

Note that we need different options on the Root command menu, depending on the function we have to work with.

Good things to know when working with GGB:

To obtain a new window in GGB, press Control N or click on the File menu and select new.

To deactivate (or reactivate) an equation or value in the algebra window, click on the blue (or open) circle to the left of the item.

If you made an error entering the function, right-click on the function in the algebra window and then select Object Properties. The right hand side of the function is shown on the Value line. You can place your cursor on that line and use the left and right arrow keys to move the cursor to the location of the error. Then backspace or use the delete key to delete the error and use your keyboard to correct it.

POPPER 2: C

The relative extrema of a function are the high points and low points of the graph of a function, when compared to other points that are close to the relative extremum.   A relative maximum will be higher than the points near it, and a relative minimum will be lower than the points near it.
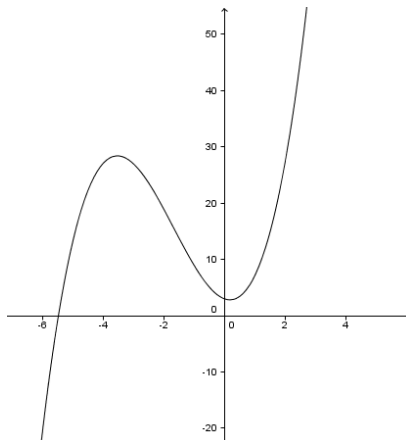
GGB will help you find these points.

Example 16: Suppose $g(x) = x^3 + 5x^2 - 2x + 3$. Find any relative extrema.

> Is this a polynomial function?

Step 1: Enter the function in the input line. Graph and resize, if needed.

Step 2: Find the relative extrema.

> What's the command?

Example 17: Suppose $h(x) = x - 3x^{\frac{1}{3}}$. Find any relative extrema.

        Is this a polynomial function?

Step 1: Enter the function in the input line. Graph and resize, if needed.

Step 2: Find the relative extrema.

        What's the command?

This is similar to the method used on a graphing calculator to find maxima and minima.

You can find the point(s) where two functions intersect by using the Intersect command in GGB.

Example 18: Find any points where $f(x) = 1.45x^2 - 7.2x - 1.6$ and $g(x) = 2.84x - 1.29$ intersect.

Step 1: Enter the functions one by one in the input line. Graph and resize if necessary to find all points of intersection.

Step 2: Find intersection points.

        What's the command?

Example 19: Find any points where $f(x) = x - \dfrac{6}{x}$ and $g(x) = 4e^{-2x} + \dfrac{1}{2}x$ intersect.

Step 1: Enter the functions one by one in the input line. Graph and resize if necessary to view all points of intersection.

Step 2: Find intersection points.

Does Intersect[<Object, Object>] return all points of intersection?

Why not?

What's the command?

POPPERS 3 – 5

Suppose $f(x) = x^3 - 12x^2 + 7x + 5$.

Match the quantity described in column A with the value in column B

Column A   Column B

3. A zero of the function   A. 7.6968

4. The x coordinate of the relative minimum   B. 11.1443

5. The y coordinate of the relative maximum   C. 1.0684

                                                            D. 0.3032

                                                            E. 6.4071

Lesson 3: Regressions Using GeoGebra

In this course, you will frequently be given raw data about a quantity, rather than a function. If this is the case, you will need to have a method for finding a function that "fits" the data that is given – that is, a function that passes through or passes close to many or most of the points of data that are given. These equations are called **regression equations**. You'll be able to use GGB to find these. The process involves several steps. Here's an example:

Example 1: Suppose you are given the data shown in the table below.

x	0	1	2	3	4	5
y	1.6	2.5	2.9	4.1	3.7	4.2

Find a linear regression model that fits the data.

Step 1: Create a table of values in the spreadsheet view of GGB.

Step 2: Create a list of ordered pairs in Column C of the spreadsheet.

Step 3: Highlight ordered pairs and create a list using the list icon in spreadsheet mode.

Step 4: Find a linear regression model (the FitPoly[<List of Points>, <Degree of Polynomial>] command, with list1 and degree 1).

You can find other regression models, as well. Using the FitPoly command, you can find polynomial models of any degree. You can also use FitExp to find exponential regression models, FitPow to find power regression models, FitLogistic to find logistic regression models.

Example 2: Using the data from Example 1, find a cubic regression model and an exponential regression model.

Cubic regression model:

Exponential regression model:

Popper 3

You will also be asked to find a value for regression models called r^2 or R^2. These values are measures of the "goodness of fit" for a regression model, and will be between 0 and 1. The closer the value is to 1, the better the linear regression model predicts the trend of the given data. The closer it is to 0, the less useful it will be in predicting future values. There are differences between the two values, in terms of how they are computed. For our purposes in this class, they will give us a piece of information for determining how well a regression equation fits the underlying data.

GGB will compute this value for you if you specify the list of points to use and the name of the regression model. The command is "RSquare[<List of points>, <Function>].

Example 3: Using the data from Example 1 and the linear, cubic and exponential regression models that you found in Examples 1 and 2, find values for r^2 or R^2 for each of the three regression models.

Linear regression model: $r^2 = 0.8714$

Cubic regression model: $R^2 = 0.9336$

Exponential regression model: $R^2 = 0.7766$

POPPER 3, problem 6: Use the value feature to find $f(2.76)$ for $y = -2x^2 + 5x + 3$.

A. 15648   B. 32.0352   C. -26.0352

Example 4: Make a scatter plot of these data. Then find a quadratic regression function that fits the data. Find the value for R^2.

$$f(x) = ax^2 + bx + c$$

Polynomial degree 2

x	-4	0	1	2	5	7	8	10	12
y	33	5	3	3	15	33	45	75	113

What are the steps needed to do this?

What is the command to find the regression model?

$$fitpoly[list1, 2]$$

$$f(x) = x^2 - 3x + 5$$

What is the command to find R^2?

$$rsquare[list1, f] = 1$$

Try it on your own!!

Use the data in table of values given below to find a quartic (fourth degree polynomial) regression and find the value for R^2.

x	1	4	7	11	14	19
y	14	17	21	16	23	21

Example 5: Suppose that we know the revenues of a company each year since 2005. This information is given in the table below:

(handwritten above table: O 1 2 3 4 5   7)
(handwritten: 2012)

year	2005	2006	2007	2008	2009	2010
revenues (in millions of dollars)	2.1	2.8	3.4	4.6	7.9	11.2

Before starting this problem, rescale the data so that the year 2005 corresponds to $x = 0$. Why would we want to do this?

(handwritten) ⮡ smaller values to work with!!

A. Create a scatterplot and determine which of the regression models are good candidates for this data.   *(handwritten)* Poly deg 2,3,4, exp.

B. Find regression models for each, and find the related values for r^2 or R^2.

- $f(x) = 0.425\,x^2 - 0.3536\,x + 2.3214$ *(handwritten)* $r^2 = 0.992$
- $g(x) = 0.0463\,x^3 + 0.0778\,x^2 + 0.2807\,x + 2.1825$ *(handwritten)* $r^2 = 0.9943$
- $h(x) = -0.0583\,x^4 + 0.6296\,x^3 - 1.7139\,x^2 + 1.9474\,x + 2.0825$ *(handwritten)* $r^2 = 0.9986$
- $p(x) = 1.9328\,e^{0.3367x}$ *(handwritten)* $r^2 = 0.9752$

C. Which model would be the best one to use? Why?

(handwritten) cubic!.   doesn't go to zero when t = 7!!

D. Use that model to predict revenues in 2012.

(handwritten) ℮ [$23.8 million]

(handwritten) g(7)

POPPER 3, question 7:   Using the data in the table, create a list in GGB and find a quadratic regression model:

x	1	3	5	7
y	12	19	26	31

    A. $f(x) = -0.125x^2 - 4.2x + 7.825$

    B. $f(x) = 0.125x^2 + 4.2x + 7.825$

    C. $f(x) = -0.125x^2 + 4.2x + 7.825$

    D. $f(x) = -0.125x^2 + 4.2x - 7.825$

POPPER 3, question 8:   Use the regression model from question 7 to find the function value when x = 15.

    A.   47.2   B. 98.95   C. 42.7   D. 99.85

POPPER 3, question 9: Find the value for R^2 for the quadratic regression model using the data for question 7.

    A.   0.7825   B. 0.125   C. 0.999   D. 0.925

Example 6: The average value of a single family residence (in thousands of dollars) during the 1970s is given by this table:

(handwritten above table: 0   2   4   6   8   18)
(handwritten above 18: 1989)

year	1971	1973	1975	1977	1979
value	42	51	63	77	93

    A. Enter the data into GGB and draw a scatterplot (what should we do first?).

(handwritten: can't have x = 0)

    B. Find linear, quadratic, cubic, quartic, exponential, power and logistic regression models. Find the value for r^2 or R^2 for each model.

(handwritten: $f(x) = ax^b$)

(handwritten number line: 1 3 5 7 9 / 42 51 63 77 93)

    C. Which regression model do you think fits the data the best? Why?

    D. Use the regression model you chose in part C to predict average value of a single family home in 1995.

(handwritten: 1971   1995)

(handwritten: power)

(handwritten: $f(25)$)

Try it!!

The value of a mutual fund (in millions of dollars) over a twenty year period is given below.

Year	1990	1995	2000	2005	2010
Value	326	409	421	452	455

Find the following regression equations which will give the value of the fund t years since 1990: linear, quadratic, cubic, exponential, logistic.

(Remember to rescale the data with $t = 0$ corresponding to 1990, and note that the years are not sequential. So $t = ?$ corresponds to 1995.)

Also find the r^2 or R^2 value for each regression.

Which model has the largest r^2 or R^2 value?

Use the model with the largest r^2 or R^2 value to predict the value of the fund in 2015.

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$\boxminus$ Dependent Objects
$\circ$ b = 0.8287
$\circ$ c = 0.9586
$\circ$ d = 0.9755
$\circ$ e = 0.794
$\bullet$ f(x) = 6.02x + 352.4
$\bullet$ $g(x) = -0.4029\,x^2 + 14.0771\,x + 332.2571$
$\bullet$ $h(x) = 0.0287\,x^3 - 1.2629\,x^2 + 20.2405\,x + 327.9571$
$\circ$ i = 0.9745
$\bullet$ $list1 = \{(0, 326), (5, 409), (10, 421), (15, 452), (20, 455)\}$
$\bullet$ $p(x) = 351.4585\,e^{0.0153x}$
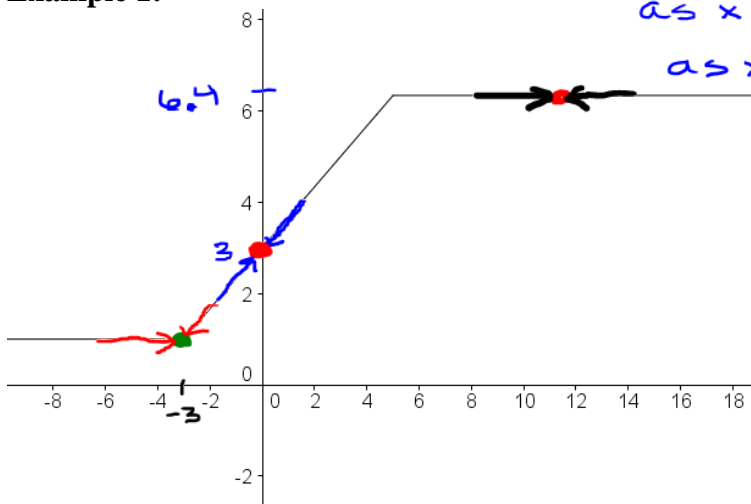$\bullet$ $q(x) = \dfrac{455.2695}{1 + 0.3881\,e^{-0.2086x}}$

Values for r^2/R^2 are given in the same order as the functions.

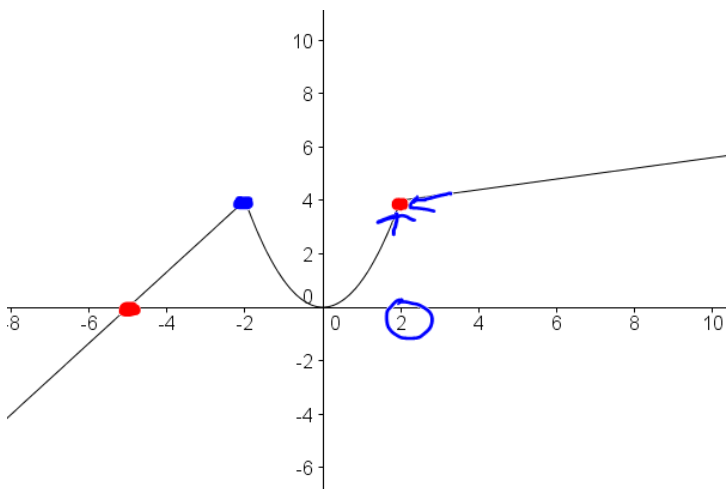**Cubic has largest R^2 value. h(25) = 492.6. $492.6 million.**

Lesson 4: Limits

If the *y*-value is approaching a specific number, then we can state the limit of the function as *x* gets close to the target number.

Example 1:

as $x \to 0$, $y \to 3$

as $x \to -3$, $y \to 1$

as $x \to 12$, $y \to 6.4$
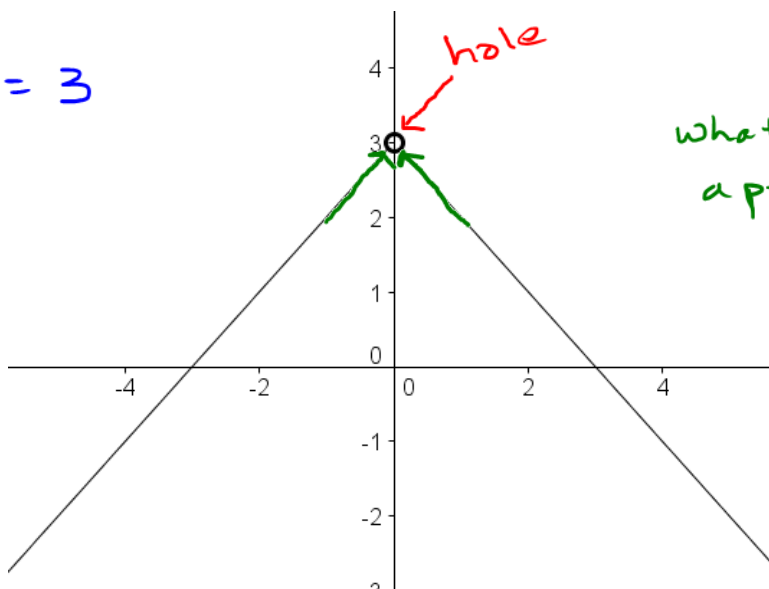
Example 2:

$\lim\limits_{x \to 2} f(x) = 4$

$\lim\limits_{x \to -5} f(x) = 0$

$\lim\limits_{x \to -2} f(x) = 4$

It does not matter whether or not the *x* value ever reaches the target number. It might, or it might not!
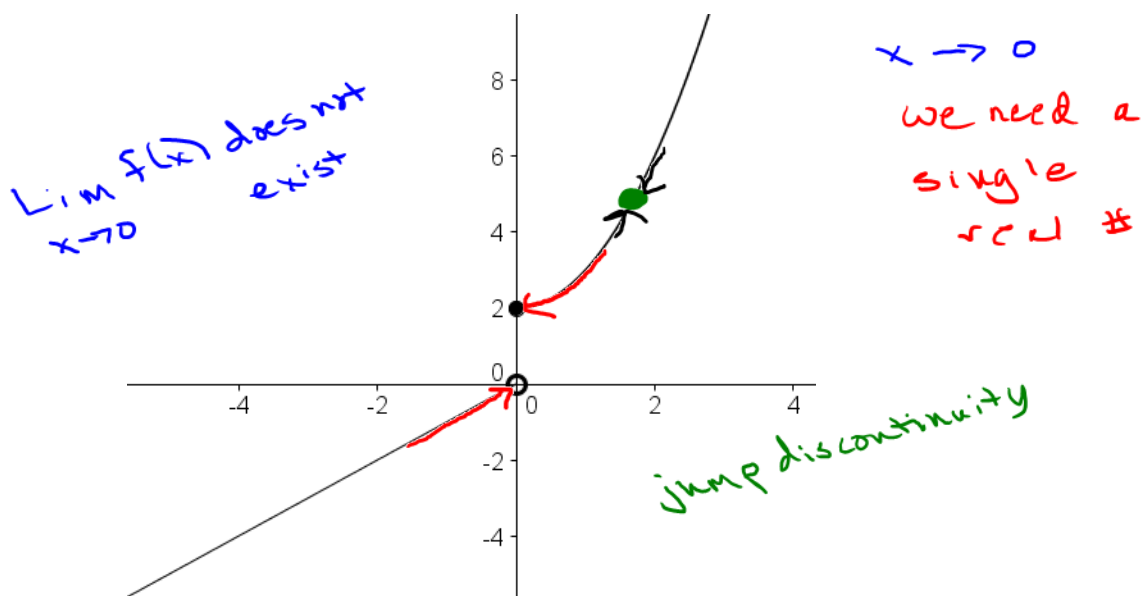
Example 3:

$$\lim_{x \to 0} f(x) = 3$$

hole

what is y
approaching?
$$y \to 3$$

When can a limit fail to exist?

We will look at two cases where a limit fails to exist (note: there are more, but some are beyond the scope of this course).

Case 1: The *y* value approaches one number from numbers smaller than the target number and it approaches a second number from numbers larger than the target number:
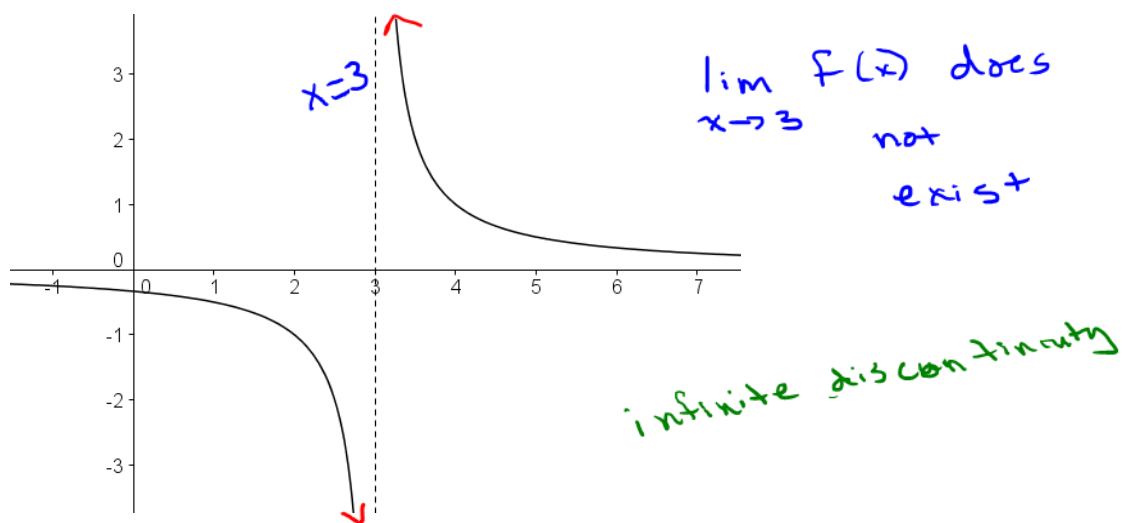
$$\lim_{x \to 0} f(x) \text{ does not exist}$$

$$x \to 0$$
we need a single real #

jump discontinuity

Case 2: At the target number for the *x*-value, the graph of the function has an asymptote.

x=3

$\lim_{x \to 3} f(x)$ does not exist

infinite discontinuity

For either of these two cases, we would say that the limit as *x* approaches the target number "does not exist."

Definition:

We say that a function *f* has **limit** *L* as *x* approaches the target number *a*, written

$$\lim_{x \to a} f(x) = L$$

if the value *f(x)* can be made as close to the number *L* as we please by taking *x* sufficiently close to (but not equal to) *a*.

Note that *L* is a single real number.

POPPER 3, question 10: D   open!

Evaluating Limits

Most often, we will not have a graph given, and will need to find a limit by algebraic methods.

Substitution

To use the first method, we will need to apply several properties of limits.

Properties of limits:

Suppose $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$. Then,

1. $\lim_{x \to a} [f(x)]^r = [\lim_{x \to a} f(x)]^r = L^r$ for any real number r.

2. $\lim_{x \to a} cf(x) = c \lim_{x \to a} f(x) = cL$ for any real number c.

3. $\lim_{x \to a} \left[f(x) \pm g(x) \right] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = L \pm M$

4. $\lim_{x \to a} [f(x)g(x)] = [\lim_{x \to a} f(x)][\lim_{x \to a} g(x)] = LM$.

5. $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \dfrac{L}{M}$, provided $M \neq 0$.

We'll use these properties to evaluate limits.

Example 3: Suppose $f(x) = x^3 - 4$. Find $\lim_{x \to 3} f(x)$.

$$\lim_{x \to 3} (x^3 - 4) = \lim_{x \to 3} x^3 - \lim_{x \to 3} 4$$

$$= \left[\lim_{x \to 3} x \right]^3 - \lim_{x \to 3} 4$$

$$3^3 - 4 = 27 - 4 = 23$$

$$\lim_{x \to 3} (x^3 - 4) = 3^3 - 4 = 27 - 4 = 23$$

Example 4: Evaluate: $\lim\limits_{x \to 0} \dfrac{4x+5}{x+3}$ $= \dfrac{4(0)+5}{0+3} = \dfrac{5}{3}$

Example 5: Evaluate: $\lim\limits_{x \to 8} 5x^{\frac{2}{3}}$ $= 5 \cdot 8^{2/3} = 5 \cdot 4 = 20$

Example 6: Evaluate: $\lim\limits_{x \to 3}\left(4x^2 - 7x + 2\right)$ $= 4(3)^2 - 7(3) + 2$

$$= 4 \cdot 9 - 21 + 2$$

$$= 36 - 21 + 2 = 17$$

What do you do when substitution gives you a value in the form $\dfrac{k}{0}$, where k is any non-zero real number?

Example 7: Evaluate: $\lim\limits_{x \to 4} \dfrac{x+8}{x-4}.$ $= \dfrac{4+8}{4-4} = \dfrac{12}{0}$

lim dne