$f(x) = x^2 - 3x + 4$ $\quad [0, 4] \quad$ $\underline{n = 4}$

① $\Delta x = \frac{b-a}{n} = \frac{4-0}{4} = \frac{4}{4} = 1$

② $[0, 1]\ [1, 2]\ [2, 3]\ [3, 4]$

left $\quad A \approx 1[f(0) + f(1) + f(2) + f(3)] = 1[4+2+2+4]$
$= 12$

right $\quad A \approx 1[f(1) + f(2) + f(3)] = 1[2+2+4+8] = 16$

midpts $\quad .5,\ 1.5,\ 2.5,\ 3.5$

$A \approx 1[f(.5) + f(1.5) + f(2.5) + f(3.5)]$

$1[2.75 + 1.75 + 2.75 + 5.75] = 13$

Use GGB

RectangleSum[f, 0, 4, 4, ~~]

left endpts = 12

right end pts = 16

mid pts = 13

Math 1314
Lesson 17: Riemann Sums Using GeoGebra; Definite Integrals

Now suppose the function and interval we wish to work with are not so pretty. You would not want to work this problem by hand:

Example 1: Approximate the area between x axis and the graph of $f(x) = 0.3x^3 - 0.807x^2 - 3.252x + 6.717$ on [-2.82, 1.33] with 10 rectangles and left endpoints.

We can work Riemann sum problems using GeoGebra. The command is RectangleSum, and you'll need to fill in the function name, start value, end value, number of rectangles and the position to use for function evaluation. We will limit the position of 0, 0.5 and 1, where 0 corresponds to left endpoints, 0.5 corresponds to midpoints and 1 corresponds to right endpoints.

Now let's try Example 1 using GGB.

Example 2: Approximate the area between x axis and the graph of $f(x) = 0.3x^3 - 0.807x^2 - 3.252x + 6.717$ on [-2.82, 1.33] with

A. 50 rectangles and midpoints endpoints.

B. 10 rectangles and right endpoints.

GGB

Example 3: Approximate the area between the x axis and the function

$f(x)=\dfrac{15}{0.083x^2+19.17x+1}$ on [0.075, 8.21] using

.5

50%

[ ]

0% 100%

0 1

A. 12 rectangles and right endpoints.

RectangleSum [f, 0.075, 8.21, 12, 1] = 2.2007

B. 29 rectangles and midpoints.

Rectangle Sum [f, 0.075, 8.21, 29, .5] = 3.1351

POPPER 11, question 1:

$\Delta x = \dfrac{b-a}{n}$

Suppose you want to approximate the area between a function and the x axis on the interval [0, 6] using Riemann sums and three sub-intervals. What is Δx?

A. 6 B. 2 C. 3 D. 0 E. 1

POPPER 11, question 2:

Suppose you want to approximate the area between a function and the x axis on the interval [0, 6] using Riemann sums and three sub-intervals. Which of these is one of the sub-intervals you would use?

A. [0, 3] B. [1, 3] C. [2, 4] D. [5, 6]

POPPER 11, question 3:

Suppose you want to approximate the area between the graph of $f(x)=x^2+3x+4$ and the x axis on the interval [0, 6] using Riemann sums, left end points and 3 subintervals. Which of these is NOT one of the function values you would need to use to approximate the area?

A. 58 B. 4 C. 32 D. 14

[0, a] [a, b] [b, c] .... [n, 6]

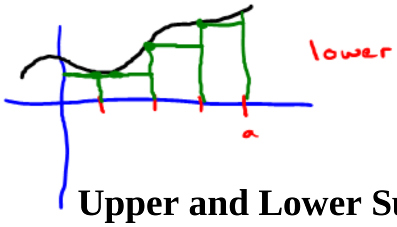
f(0) = ___

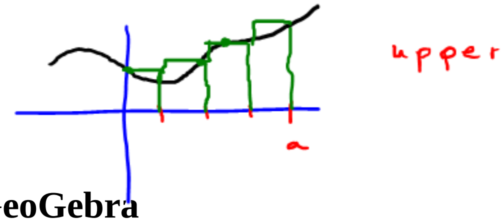
f(a) = ___

f(b) = ___

⋮

f(n) = ___





Upper and Lower Sums Using GeoGebra

You can also find a related quantity using GeoGebra, the upper sum and/or the lower sum. Rather than always using the left endpoint, the right endpoint or the midpoint of the interval to find the height of the rectangle, the upper sum uses the biggest *y* value on each interval as the height of the rectangle and the lower sum uses the smallest *y* value on each interval as the height of the rectangle, no matter where on the interval that value occurs.

The commands are UpperSum and LowerSum. For both commands, you need to fill in the function name, the start value, the end value and the number of rectangles.

Example 4: Approximate the area between the x axis and the function $f(x)=\frac{1}{2}e^{x}+2$ on the interval [-2, 6] using 45 rectangles and

(A) Upper sums

UpperSum[f, -2, 6, 45] = 236.1017

(B) Lower sums

LowerSum[f, -2, 6, 45] = 200.2534

Example 5: Approximate the area between the x axis and the function $f(x)=x\sqrt{0.3x^{2}+7}$ on the interval [1.81, 4.93] using

(A) Upper sums and 20 rectangles

UpperSum[g, 1.81, 4.93, 20] = 36.0299

(B) Lower sums and 20 rectangles

LowerSum[g, 1.81, 4.93, 20] = 33.9203

(C) Upper sums and 50 rectangles 35.3927

Approximate the area between the x axis and the function $f(x) = x\sqrt{0.3x^2 + 7}$ on the interval [1.81, 4.93] using

POPPER 11, question 4:

(D) Lower sums and 50 rectangles

A. 35.3927 B. 34.8852 C. 34.5488 D. 34.9695

↑ Approximations

Exact area ↓

The Definite Integral

Let f be defined on $[a, b]$. If $\lim_{n\to\infty}\Big(\underbrace{[f(x_1)+f(x_2)+\ldots+f(x_n)]\Delta x}_{\text{Riemann sum}}\Big)$ exists for all choices of representative points in the n subintervals of $[a, b]$ of equal width $\Delta x = \dfrac{b-a}{n}$, then this limit is called the definite integral of f from a to b. The definite integral is noted by $\int_a^b f(x)dx = \lim_{n\to\infty}\big([f(x_1)+f(x_2)+\ldots+f(x_n)]\Delta x\big)$. The number a is called the lower limit of integration and the number b is called the upper limit of integration.

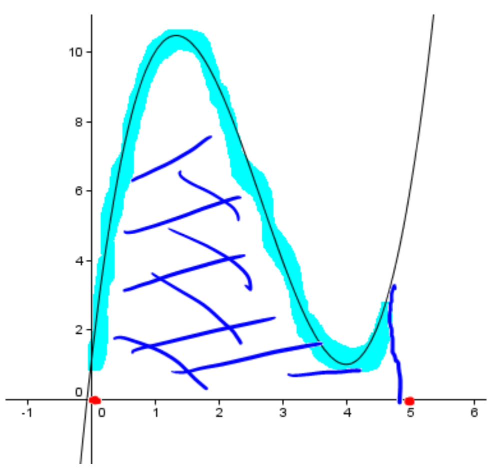
$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\big([f(x_1)+f(x_2)+\cdots+f(x_n)]\Delta x\big)$$

definite integral

The definite integral of a nonnegative function: (doesn't dip below x axis)

[0, 5]

$\int_0^5 f(x)\,dx =$ exact area between f and x axis



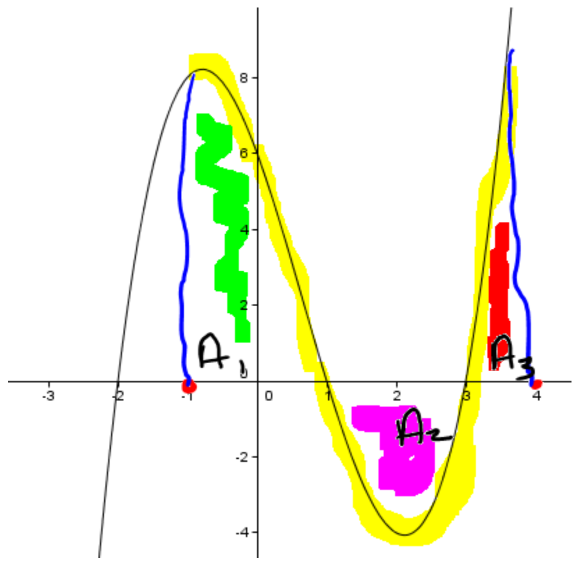
The definite integral of a <u>general function</u>: dips below x axis

$[-1, 4]$

$$\int_{-1}^{4} f(x)\,dx =$$

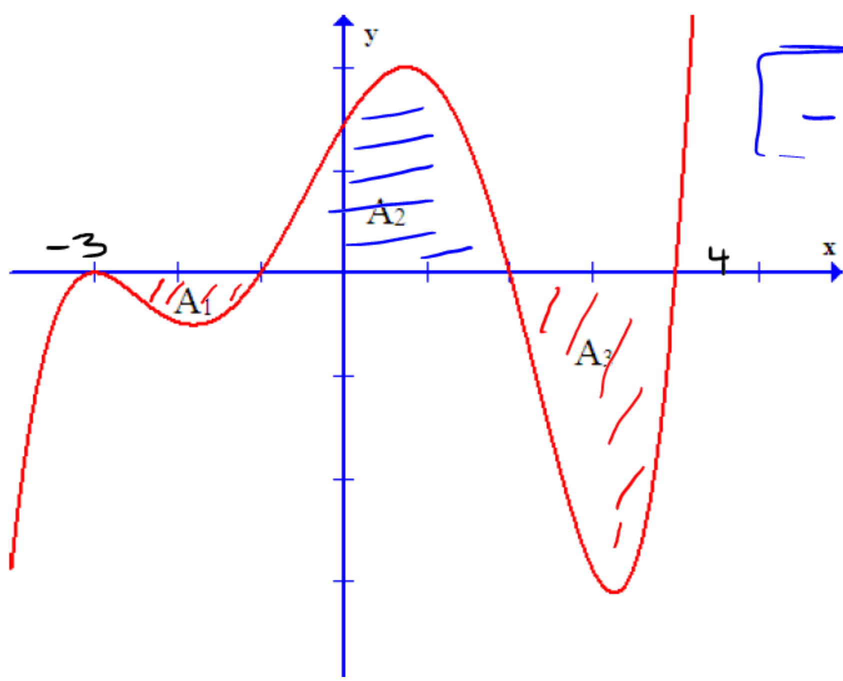
"net"

$A_1 + A_3 - A_2$



Example 6: Suppose $A_1 = 12$, $A_2 = 150$ and $A_3 = 90$, find $\int_{-3}^{4} f(x)dx$ $= -12 + 150 - 90$

$= 48$

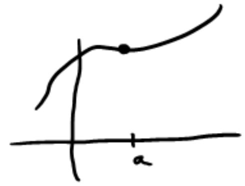


$\boxed{-A_1 + A_2 - A_3}$

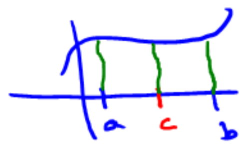
Properties of Definite Integrals

Properties of Definite Integrals

Suppose $f(x)$ and $g(x)$ are integrable functions. Then:



1. $\int_a^a f(x)\,dx = 0$
2. $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$ $\quad \int_2^3 2x^2\,dx = 2\int_2^3 x^2\,dx$
3. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ $\quad \int_4^1 f(x)\,dx = -\int_1^4 f(x)\,dx$ ***
4. $\int_a^b [f(x) \pm g(x)]\,dx = \int_b^a f(x)\,dx \pm \int_a^b g(x)\,dx$ $\quad \int_0^2 (x^2+3)\,dx = \int_0^2 x^2\,dx + \int_0^2 3\,dx$
5. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx, \quad a < c < b$



Example 7: Suppose $\int_2^5 f(x)\,dx = 22$, $\int_2^5 [f(x)]^2\,dx = 201$, $\int_2^5 g(x)\,dx = 10.5$. Find

A. $\int_2^5 [f(x)]^2\,dx + \int_2^5 f(x)\,dx - \int_2^5 g(x)\,dx = 201 + 22 - 10.5 = \boxed{212.5}$

B. $\int_2^5 4[f(x)]^2\,dx - \int_2^5 2f(x)\,dx = 4\int_2^5 [f(x)]^2\,dx - 2\int_2^5 f(x)\,dx = 4(201) - 2(22)$
$= 804 - 88$
$= 716$

C. $\int_2^2 [f(x)]^2\,dx + \int_2^2 f(x)\,dx = 0$

D. $\int_2^3 f(x)\,dx + \int_3^5 \cancel{g}\,f(x)\,dx = \int_2^5 f(x)\,dx = 22$