Math 1314
Lesson 17: Riemann Sums Using GeoGebra; Definite Integrals

Now suppose the function and interval we wish to work with are not so pretty. You would not want to work this problem by hand:

Example 1: Approximate the area between x axis and the graph of $f(x) = 0.3x^3 - 0.807x^2 - 3.252x + 6.717$ on [-2.82, 1.33] with 10 rectangles and left endpoints.

We can work Riemann sum problems using GeoGebra. The command is RectangleSum, and you'll need to fill in the function name, start value, end value, number of rectangles and the position to use for function evaluation. We will limit the position of 0, 0.5 and 1, where 0 corresponds to left endpoints, 0.5 corresponds to midpoints and 1 corresponds to right endpoints.

Now let's try Example 1 using GGB.

RectangleSum[f, start, end, # of rectangle, position]

26.8026

Example 2: Approximate the area between x axis and the graph of $f(x) = 0.3x^3 - 0.807x^2 - 3.252x + 6.717$ on [-2.82, 1.33] with

A. 50 rectangles and midpoints endpoints. 26.7597

B. 10 rectangles and right endpoints. 26.3577

POPPER 10, question 9

Use the equation and interval from Example 2 and approximate the area between the x axis and the function on the interval using

9. left endpoints and 31 rectangles

A. 26.5174 B. 26.8096 C. 26.6661 D. 26.7653

10. midpoints and 75 rectangles

A. 26.7577 B. 26.7827 C. 26.7234 D. 26.7931

Example 3: Approximate the area between the x axis and the function $f(x)=\dfrac{15}{0.083x^2+19.17x+1}$ on [0.075, 8.21] using

A. 12 rectangles and right endpoints.

B. 29 rectangles and midpoints.

Upper and Lower Sums Using GeoGebra

You can also find a related quantity using GeoGebra, the upper sum and/or the lower sum. Rather than always using the left endpoint, the right endpoint or the midpoint of the interval to find the height of the rectangle, the upper sum uses the biggest *y* value on each interval as the height of the rectangle and the lower sum uses the smallest *y* value on each interval as the height of the rectangle, no matter where on the interval that value occurs.

The commands are UpperSum and LowerSum. For both commands, you need to fill in the function name, the start value, the end value and the number of rectangles.

Example 4: Approximate the area between the x axis and the function $f\left(x\right)=\frac{1}{2}e^x+2$ on the interval [-2, 6] using 45 rectangles and

(A) Upper sums

(B) Lower sums

Example 5: Approximate the area between the x axis and the function $f(x) = x\sqrt{0.3x^2 + 7}$ on the interval [1.81, 4.93] using

(A) Upper sums and 20 rectangles

(B) Lower sums and 20 rectangles

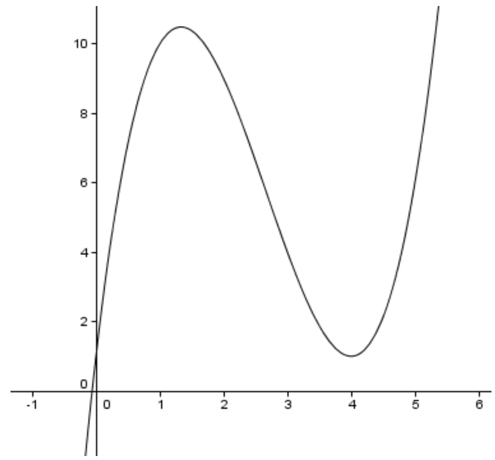
(C) Upper sums and 50 rectangles

(D) Lower sums and 50 rectangles

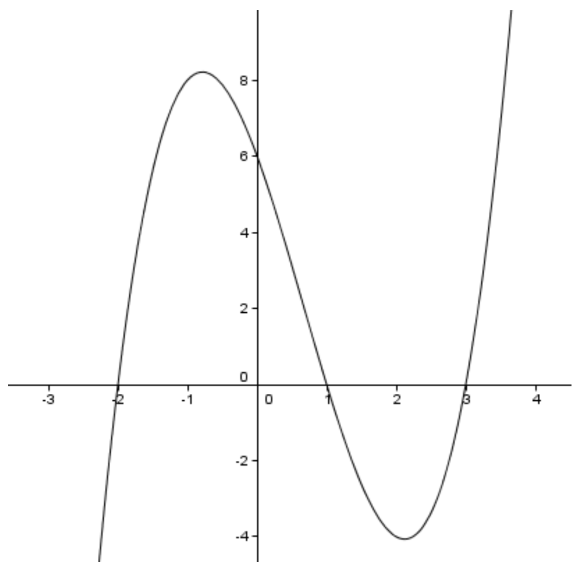
The Definite Integral

Let *f* be defined on [*a*, *b*]. If $\lim_{n\to\infty}\left([f(x_1) + f(x_2) + \ldots + f(x_n)]\Delta x\right)$ exists for all choices of representative points in the *n* subintervals of [*a*, *b*] of equal width $\Delta x = \dfrac{b-a}{n}$, then this limit is called the definite integral of *f* from *a* to *b*. The definite integral is noted by $\int_a^b f(x)dx = \lim_{n\to\infty}\left(\left[f(x_1) + f(x_2) + ... + f(x_n)\right]\Delta x\right)$. The number *a* is called the lower limit of integration and the number *b* is called the upper limit of integration.

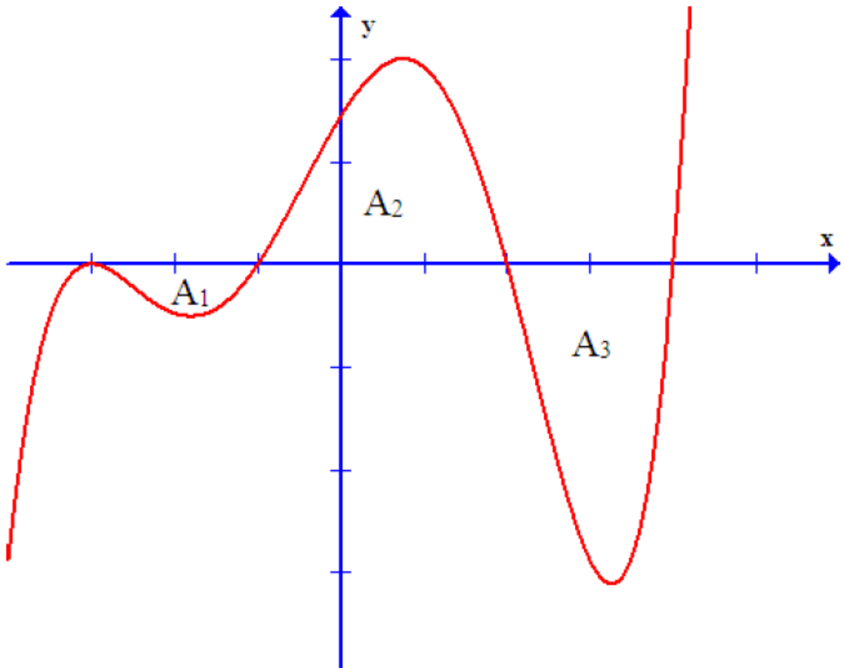
The definite integral of a nonnegative function:



The definite integral of a general function:



Example 6: Suppose $A_1 = 12$, $A_2 = 150$ and $A_3 = 90$, find $\int_{-3}^{4} f(x)dx$



Properties of Definite Integrals

Suppose $f(x)$ and $g(x)$ are integrable functions. Then:

1. $\int_a^a f(x)\,dx = 0$
2. $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$
3. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
4. $\int_a^b \left[f(x) \pm g(x)\right]dx = \int_b^a f(x)\,dx \pm \int_a^b g(x)\,dx$
5. $\int_a^b f(x)\,dx = \int_c^a f(x)\,dx + \int_c^b f(x)\,dx, \quad a < c < b$

Example 7: Suppose $\int_2^5 f(x)\,dx = 22, \int_2^5 \left[f(x)\right]^2 dx = 201, \int_2^5 g(x)\,dx = 10.5$. Find

A. $\int_2^5 [f(x)]^2 dx + \int_2^5 f(x)\,dx - \int_2^5 g(x)\,dx$

B. $\int_2^5 4[f(x)]^2 dx - \int_2^5 2f(x)\,dx$

C. $\int_2^2 [f(x)]^2 dx + \int_2^2 f(x)\,dx$

D. $\int_2^3 f(x)\,dx + \int_3^5 g(x)\,dx$