Math 1314
Lesson 14 - Optimization

When you optimize something, you make it as large as possible or as small as possible under certain stated conditions. Business owners wish to make revenues and profits as large as possible, while keeping costs as small as possible. There are many other applications as well. In this unit, we'll start by looking at optimization generally; then we'll move on to some applications.

Absolute Extrema

Definition: If $f(x) \le f(c)$ for all x in the domain of f, then $f(c)$ is called the **absolute maximum value** of f. If $f(x) \ge f(c)$ for all x in the domain of f, then $f(c)$ is called the **absolute minimum value** of f.

So the y value of the point that has the biggest y value in the interval of interest is the absolute maximum value. The y value of the point that has the smaller y value in the interval of interest is the absolute minimum value.

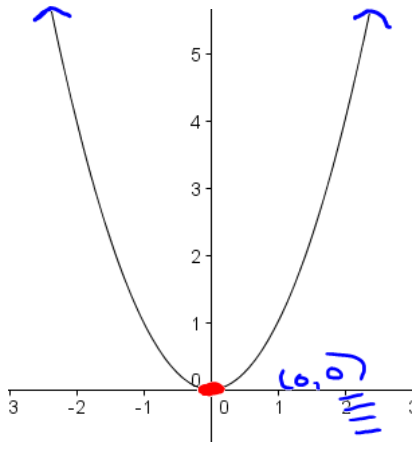
When you are looking at a graph of the function on its entire domain, sometimes you have an absolute maximum but no absolute minimum; sometimes you have an absolute minimum but no absolute maximum; sometimes you have both; sometimes you have neither.

Sometimes, you'll be given a closed interval and asked to find the absolute extrema on that interval. In that case, you are only interested in the behavior of the function on that interval. If you have a continuous function on a closed interval, you are guaranteed to have both an absolute maximum and an absolute minimum value.

We can find absolute extrema by looking at a graph of a function.

Example 1: State the absolute maximum and/or absolute minimum value(s).

absolute minimum
(bottom point)
$f(0) = 0$
abs. min value



abs. extreme refer to y value

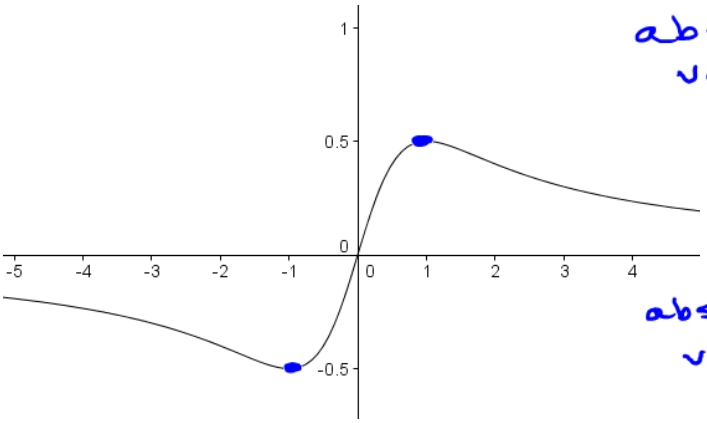
Example 2: State the absolute maximum and/or absolute minimum value(s).

absolute max

$f(7) = 0.5$

absolute min

$f(-7) = -0.5$

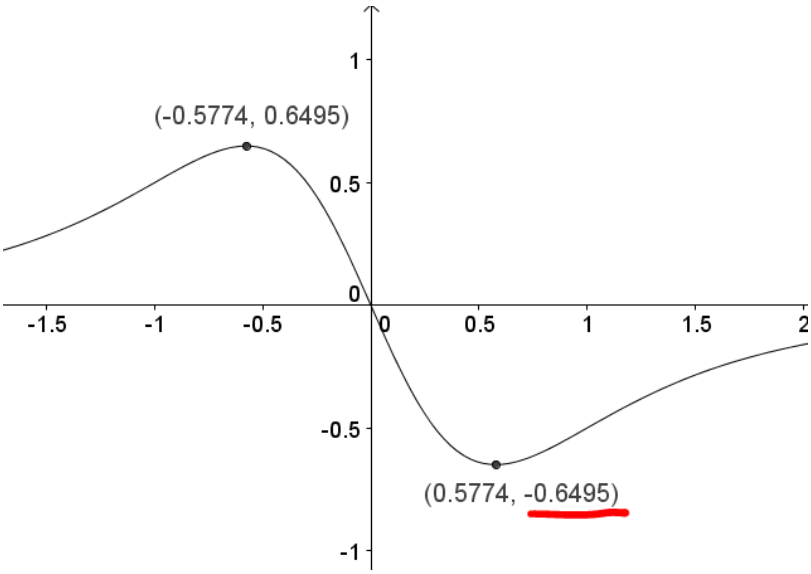


abs max value = 0.5

abs min value = -0.5

POPPER 9, question 7:

Find the absolute minimum given the graph.



A. -0.5774 B. 0.5774 C. 0.6495 D. -0.6495

Example 3: Find the absolute maximum and absolute minimum values of the function $f(x) = 3 + 4x^2 - x^4$.

no absolute minimum

absolute max value at $y = 7$

Example 4: Find the absolute maximum and absolute minimum values of the function $f(x) = \sqrt{4 - x^2}$.

abs max is $y = 2$

abs max is $f(0) = 2$

abs min is $y = 0$

abs min is $f(-2) = f(2) = 0$

Example 5: Find the absolute extrema of the function $f(x) = x^2 - 5x + 6$ on $[1, 3]$.

continuous closed interval

guaranteed to have both an abs max and abs min

will occur at c.n. or at end point

① find critical numbers

$f'(x) = 2x - 5$ $\quad 2x - 5 = 0$ $\quad 2x = 5$ $\quad x = 5/2$

② test c.n. and endpts

x	f(x)
1	2
2.5	−0.25
3	0

abs min at $f(2.5) = -0.25$

abs max at $f(1) = 2$

Example 6: Find the absolute extrema of the function $f(x) = \sqrt{x}\left(x^3 - 4\right)^2$ on $[0.5, 1]$.

abs. min $\quad \text{Min}[f, 0.5, 1]$

$f(1) = 9$

abs max $\quad \text{Max}[f, 0.5, 1]$

$f(0.6751) = 11.2016$

Example 7: An apartment complex estimates that the revenues realized from renting out x of its 100 one-bedroom apartments can be modeled by the function $R(x) = -12x^2 + 2112x$. How many one bedroom apartments should be rented to maximize the revenue? What is the maximum revenue?

Maximize $R(x) = -12x^2 + 2112x$

$[0, 100]$

Max $[R, 0, 100] \longrightarrow (88, 92928)$

Rent 88 apts

Max revenue $92928

Example 8: The height of a rocket *t* seconds after launch is given by the function $h(t) = -\frac{1}{3}t^3 + 6t^2 + 12$, where $h(t)$ is given in feet. Find the time *t* when the rocket reaches its maximum height. What is the maximum height of the rocket?

max $h(t)$ on $(-\infty, \infty)$

Extremum [polynomial] $\longrightarrow$ $(12, 300)$

12 sec

max ht 300 ft

$x^2 = 25$

$x = \pm 5$

POPPER 9, question 8:

Suppose $f(x) = 2x^3 - 24x + 5$. Find any critical numbers.

A. 2 B. 0, 2 C. -2, 2 D. 0, 4 E. -4, 0, 4

$f'(x) = 6x^2 - 24$

$6x^2 - 24 = 0$

$6x^2 = 24$

$x^2 = 4$

POPPER 9, question 9:

Suppose $f(x) = 2x^3 - 24x + 5$. Find the absolute maximum value on the interval [0, 4].

A. 5 B. 27 C. 4 D. 0 E. 37

POPPER 9, question 10:

Does $f(x) = 12 - 3x - 2x^2$ have an absolute maximum value, an absolute minimum value, both or neither?

A. Absolute max B. Absolute min C. Both D. Neither