Midterm 1 – Friday / Saturday
Lessons 1-8

Math 1314
Lesson 10: Elasticity of Demand

Suppose you owned a small business and needed to make some decisions about the pricing of your products. It would be helpful to know what effect a small change in price would have on the demand for your product. If a price change will have no real change on demand for the product, it might make good sense to raise the price. However, if a price increase will cause a big drop in demand, then it may not be a good idea to raise prices.

There is a measure of the responsiveness of demand for an object to a change in its price: *elasticity of demand*. This is defined as

$$\frac{\text{percentage change in demand}}{\text{percentage change in price}}.$$

formula needed

To develop this formula, we'll start by solving our demand function for *x*, so that we have a function $x = f(p)$. Then we have a demand function in terms of price. If we increase the price by *h* dollars, then the price is $p+h$ and the quantity demanded is $f(p+h)$.

The percentage change in demand is $100 \cdot \frac{f(p+h)-f(p)}{f(p)}$ and the percentage change in price is $100 \cdot \frac{h}{p}$.

If we compute the ratio given above, we have $\dfrac{100 \cdot \frac{f(p+h)-f(p)}{f(p)}}{100 \cdot \frac{h}{p}}$.

We can simplify this to

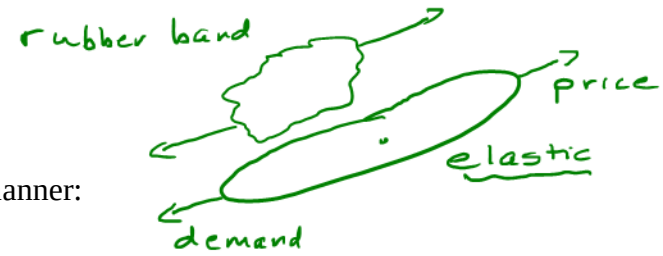
$$\frac{p}{f(p)}\left[\frac{f(p+h)-f(p)}{h}\right].$$

For small values of h, $\frac{f(p+h)-f(p)}{h} \approx f'(p)$, so we have $\frac{p \cdot f'(p)}{f(p)}$.

This quantity is almost always negative, and it is easier to work with a positive value, so we define the negative of this ratio to be the elasticity of demand.

Then

$E(p) = -\frac{p \cdot f'(p)}{f(p)}$, where p is price in dollars, $f(p)$ is the demand function and $f(p)$ is differentiable when $x = p$.



Revenue responds to elasticity in the following manner:

If demand is elastic at p, then
- An increase in unit price will cause revenue to decrease or
- A decrease in unit price will cause revenue to increase

opposite directions

If demand is unitary at p, then
- An increase in unit price will cause the revenue to stay about the same.

If demand is inelastic at p, then
- An increase in the unit price will cause revenue to increase
- A decrease in unit price will case revenue to decrease.

same direction

We have these generalizations about elasticity of demand:

Demand is said to be elastic if $E(p) > 1$.
Demand is said to be unitary if $E(p) = 1$.
Demand is said to be inelastic if $E(p) < 1$.

$E(p)$ is a number

So, if demand is elastic, then the change in revenue and the change in price will move in opposite directions.

If demand is inelastic, then the change in revenue and the change in price will move in the same direction.

Example 1: Find $E(p)$ for the demand function $x + 2p - 15 = 0$ and determine if demand is elastic, inelastic, or unitary when $p = 4$.

(b)

$$E(p) = \frac{-p f'(p)}{f(p)}$$

$x + 2p - 15 = 0$

① solve demand equation for x

$x + 2p - 15 = 0$

$f(p) = x = 15 - 2p$

② $f'(p) = -2$

③ $E(p) = \frac{-p(-2)}{15 - 2p}$

$E(p) = \frac{2p}{15 - 2p}$

(b)

$E(4) = \frac{2(4)}{15 - 2(4)}$

$= \frac{8}{15 - 8} = \frac{8}{7}$

$E(4) = \frac{8}{7} > 1$

demand is elastic

at a price of \$4, an increase in price results in a decrease in demand

Example 2: Suppose the demand function for a product is given by $p = -0.02x + 400$. This function gives the unit price in dollars when x units are demanded.

(A) Find the elasticity of demand.

Solve p for x $\quad p = -0.02x + 400$

$\frac{1p}{-0.02} - \frac{400}{-0.02}$ $\quad \frac{1p - 400}{-0.02} = \frac{-0.02x}{-0.02}$

$-50p + 20000 = x = f(p)$

Find $f'(p)$ $\quad f'(p) = -50$

Find $E(p)$ $\quad E(p) = \frac{-p(-50)}{20000 - 50p} = \frac{50p}{20000 - 50p}$

(B) Find $E(100)$ and interpret the results. If the unit price is \$100, will raising the price result in an increase in revenues or a decrease in revenues?

$E(100) = \frac{50(100)}{20000 - 50(100)} = \frac{5000}{15000} = \frac{1}{3}$

demand is inelastic

increase in revenues (demand / revenues will move in same directions)

(C) Find $E(300)$ and interpret the results. If the unit price is \$300, will raising the price result in an increase in revenues or a decrease in revenues?

$E(300) = \frac{50(300)}{20000 - 50(300)} = 3$

demand is elastic

decrease in revenues (demand / revenues will move in opposite directions)

What else can $E(p)$ tell you?

Example 4: If $E(p)=\frac{1}{2}$ when $p=250$, what effect will a 1% increase in price have on revenue?

demand is inelastic

if price increases, revenue increases too

a 1% increase in price will result in a $\frac{1}{2}$% increase in revenues

Example 5: If $E(p)=\frac{3}{2}$ when $p=250$, what effect will a 1% increase in price have on revenue?

>1

demand is elastic

when price increases revenue will decrease

a 1% increase in price will result in a 1.5% decrease in revenue.