28 questions; 25 mc, 3 FR

Lessons 18-24

@ 1 hour cumulative

@ 50 min material since last test

1hr 50 min to take

Math 1314 ONLINE
Final Exam Review

Review Example 1: Suppose $g(x) = x^3 - 2x^2 - 9x + 18$. Find the zeros of the function.

Root [polynomial]

$\boxed{x = -3, 2, 3}$

Review Example 2: Find any points where $f(x) = 1.45x^2 - 7.2x - 1.6$ and $g(x) = 2.84x - 1.29$ intersect.

Intersect [object, object]

f g

(-0.0307, -1.3773)

(6.9547, 18.4619)

Review Example 3: Suppose that we know the revenues of a company each year since 2005. This information is given in the table below:

	0	1	2	3	4	5
year	2005	2006	2007	2008	2009	2010
revenues (in millions of dollars)	2.1	2.8	3.4	4.6	7.9	11.2

7

2012

A. Create a scatterplot and determine which of the regression models are good candidates for this data.

Spreadsheet view; enter points; create list

cubic, ~~quartic~~, exponential

B. Find regression models for each, and find the related values for r^2 or R^2.

Fitpoly [list1, 3] $f(x) = 0.463x^3 + 0.0788x^2 + .2901x + 2.1825$

Fitexp [list1] $g(x) = 1.9328e^{0.3367x}$

RSquare [list1, f] = .9943 RSquare [list1, g] = .9752

C. Which model would be the best one to use? Why?

Either .... good R^2 .... fit data well...

cubic

D. Use that model to predict revenues in 2012.

f (7) = ______

Review Example 4: Evaluate: $\lim_{x \to 4} \frac{x+8}{x-4}$. $= \frac{4+8}{4-4} = \frac{12}{0}$

suppose $\frac{0}{12} = 0$ ✓

limit dne

Review Example 5: Evaluate: $\lim_{x \to 3} (4x^2 - 7x + 2)$ $= 4 \cdot 3^2 - 7(3) + 2$

$= 4 \cdot 9 - 21 + 2$

$= 36 - 21 + 2 = \boxed{17}$

enter into GGB

Limit[function, value]

Limit[f, 3]

Review Example 6: Determine if the function given is indeterminate. If it is, use GGB to evaluate the limit.

$$\lim_{x \to 2} \frac{x^2 + 3x - 10}{x^2 - 4}$$

① $\frac{2^2 + 3 \cdot 2 - 10}{2^2 - 4} = \frac{4 + 6 - 10}{4 - 4}$

$= \frac{0}{0}$

Indeterminate

② enter into GGB

$f(x) = \frac{x^2 + 3x - 10}{x^2 - 4}$

Limit[f, 2] = 1.75

Review Example 7: Evaluate: $\lim_{x \to \infty} \frac{5x^2 + 2x + 6}{2x^2 - 7x + 1}$ $= \boxed{\frac{5}{2}}$

EATS DC

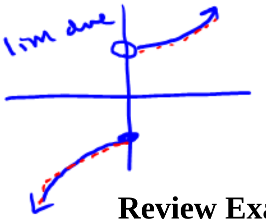
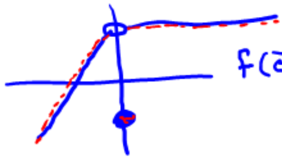
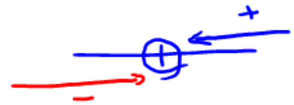
Review Example 8: Evaluate : $\lim_{x \to \infty} \frac{x - 7}{x^2 + 9x - 3}$ $= \boxed{0}$

BOBO

Review Example 9: Evaluate: $\lim_{x \to -\infty} \frac{3x^4 - 4x + 5}{2x - 7x^2}$ $\boxed{\text{dne}}$

BOT N

BOBO BOT N EATS DC

lim dne   $f(0) \neq \lim_{x\to 0} f(x)$ 

Review Example 10: Suppose $f(x) = \begin{cases} x+4, & x>2 \\ x^2-1, & x \le 2 \end{cases}$. Find $\lim_{x\to 2^-} f(x)$, $\lim_{x\to 2^+} f(x)$ and $\lim_{x\to 2} f(x)$ (if it exists).

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x^2-1) = 2^2 - 1 = 4-1 = \boxed{3}$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x+4) = 2+4 = \boxed{6}$

$\lim_{x\to 2} f(x)$ $\boxed{\text{dne}}$

continuity test

Review Example 11: Determine if $f(x) = \begin{cases} 5x-1, & x \ge 1 \\ x^2+3, & x<1 \end{cases}$ is continuous at $x = 1$.

① $f(1) = 5(1)-1 = 5-1 = 4$

② $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x^2+3) = 1^2+3 = 4$ $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (5x-1) = 5(1)-1 = 4$

$\lim_{x\to 1} f(x) = 4$

③ $f(1) = \lim_{x\to 1} f(x) = 4$

$\boxed{\text{conclusion: } f \text{ is continuous at } x=1}$

Review Example 12: Find the derivative of $f(x) = -2x^2 + 6x + 3$ using the limit definition.

MIC

$\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$f(x) = -2x^2 + 6x + 3$

$\boxed{f'(x) = -4x + 6}$

AROC

Review Example 13: Suppose $f(x) = 3x^2 - 4x - 1$. Find the average rate of change of f over the interval [2, 5].

a b

$\dfrac{f(b)-f(a)}{b-a}$

$\dfrac{f(5)-f(2)}{5-2}$

$f(5) = 3\cdot 5^2 - 4\cdot 5 - 1 = 54$

$f(2) = 3\cdot 2^2 - 4\cdot 2 - 1 = 3$

$\dfrac{54-3}{3} = \dfrac{51}{3}$

$= \boxed{17}$

Review Example 14: Find the derivative: $f(x) = 5x^4 + 3x^3 - 4 - \frac{6}{x}$.

$f(x) = 5x^4 + 3x^3 - 4 - 6x^{-1}$

$f'(x) = 20x^3 + 9x^2 - 0 + 6x^{-2} = \boxed{20x^3 + 9x^2 + \frac{6}{x^2}}$

Review Example 15: Find the numerical derivative of $f(x) = x^{\frac{3}{2}} - x$ when x = 4.

enter $f(x)$

$f'(4) = 2$

Review Example 16: Find the numerical derivative of $f(x) = \frac{x}{\sqrt{x^2+1}}$ at x = 2.

enter $f(x)$

$f'(2) = 0.0894$

Review Example 17: Write an equation of the line that is tangent to $f(x) = \frac{1}{3}x^3 - 2x^2 + 7x$ when $x = 3$.

enter $f(x)$

Tangent[3, f] => $y = 4x$

Tangent[point, function]

by hand

$f'(x) = x^2 - 4x + 7$

$m = f'(3) = 9 - 12 + 7 = 4$

$f(3) = \frac{27}{3} - 2 \cdot 9 + 21 = 9 - 18 + 21 = 12$

$m = 4 \quad (3, 12)$

$12 = 4(3) + b$

$12 = 12 + b$

$b = 0$

$\boxed{y = 4x}$

Review Example 18: A country's gross domestic product (in millions of dollars) is modeled by the function $G(t) = -2t^3 + 45t^2 + 20t + 6000$ where $0 \le t \le 11$ and $t = 0$ corresponds to the beginning of 1997.

2002 − 1997 = 5 2004 − 1997 = 7

ROC (A) At what rate was GDP changing at the beginning of 2002? At the beginning of 2004? At the beginning of 2009? 2009 − 1997 = 12

(B) What was the average rate of growth of the GDP over the period 1999 – 2004?

AROC [2, 7]

A. enter G(t)

$G'(5) = 320$ \$320 million / year

$G'(7) = 356$ \$356 million / year

$G'(12)$ (12 is not in domain; can't use the model)

B. $\frac{G(7) - G(2)}{7 - 2} = \frac{7459 - 6204}{5} = \frac{1455}{5} = 291$

\$291 million / year.

$h(t) = -16t^2 + v_0 t + h_0$ feet

→ $h(t) = -4.9t^2 + v_0 t + h_0$ meters

Review Example 19: Suppose you are standing on the top of a building that is 28 meters high. You throw a ball up into the air, with initial velocity of 10.2 meters per second. ① Write the equation that gives the height of the ball at time *t*. Then use the equation to find the velocity of the ball when $t = 2$.

① $h(t) = -4.9t^2 + 10.2t + 28$

② velocity $= h'(t)$

$h'(2) = -9.4$ $\boxed{-9.4 \text{ m/sec}}$

Review Example 20: Find all values of x for which $f'(x) = 0$: $f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 - 7x + 3$

① find $f'(x)$

② find zeros of $f'(x)$

Root[f']

$x = -6.1401,\ 1.1401$

Review Example 21: Find the value of the second derivative when x = 2:
$f(x) = 3x^4 - 5x^3 + 7x + 12$

enter $f(x)$

$f''(2) = \boxed{84}$

or

$f'(x) = 12x^3 - 15x^2 + 7$

$f''(x) = 36x^2 - 30x$

$f''(2) = 36(2)^2 - 30(2)$

$= 144 - 60$

$= \boxed{84}$

4.2

Review Example 22: Suppose a company can model its costs according to the function $C(x) = 0.000003x^3 - 0.04x^2 + 200x + 70{,}000$ where $C(x)$ is given in dollars and demand can be modeled by $p = -0.02x + 300$. ① Write the revenue function and find the smallest positive quantity for which all costs are covered.

① $R(x) = xp$

$R(x) = x(-0.02x + 300)$

② Intersect[C, R]

$x = 628.4552$

$\boxed{629}$

~~628~~ small loss

629 small profit

demand **Review Example 23:** The quantity demanded of a certain electronic device is 8000 units when the price is \$260. At a unit price of \$200, demand increases to 10,000 units. The manufacturer supply will not market any of the device at a price of \$100 or less. However for each \$50 increase in price above \$100, the manufacturer will market an additional 1000 units. Assume that both the supply equation and the demand equation are linear. Find the supply equation, the demand equation and the equilibrium quantity and price.

demand (8000, 260) (10000, 200)

Find a linear regression model for demand

$d(x) = -0.03x + 500$ Fitpoly[list1, 1]

supply (0, 100) (1000, 150)

Find a linear regression model for supply

fit poly [list 1, 1]

$s(x) = 0.05x + 100$

equilibrium quantity | price Intersect [d, s] = (5000, 300) quantity price

Marginal analysis **Review Example 24:** The weekly demand for a certain brand of DVD player is given by $p = -.02x + 300,\ 0 \le x \le 15{,}000$, where p gives the wholesale unit price in dollars and x denotes the quantity demanded. The weekly cost function associated with producing the DVD players is given by $C(x) = 0.000003x^3 - 0.04x^2 + 200x + 70{,}000$. ① Compute $C'(3000)$, $R'(3000)$ and $P'(3000)$. Interpret your results

① $C'(3000) = 41$ Actual cost to produce the 3001st item is about \$41.

② Write revenue function $R(x) = xp$

$R(x) = x(-0.02x + 300)$

$R'(3000) = 180$

Actual revenue realized on the sale of 3001st item is approx \$180

③ Write profit function

$P(x) = R - C$

$P'(3000)$ Actual profit realized on the sale of 3001st item is approx \$139

4.2

Average Cost $\bar{C}(x) = \frac{C(x)}{x}$

Average cost of producing 3000 items

Not a marginal function

$A(x) = C(x)/x \dashrightarrow A(3000)$

Review Example 25: Suppose $p = -0.04x + 150$.

(A) Find the elasticity of demand.

(B) Find $E(50)$ and interpret the results.

(C) Find $E(100)$ and interpret the results.

(D) If the unit price is \$50, will raising the price result in an increase in revenues or a decrease in revenues?

(E) If the unit price is \$100, will raising the price result in an increase in revenues or a decrease in revenues?

$f(p) = x = -12p + 12500$ $\quad f'(p) = -12$

A. Solve $p = -0.04x + 150$ for x

$E(p) = \dfrac{-p(-12)}{-12p + 12500} = \dfrac{12p}{den}$

In GGB → view
CAS
In the line marked 1 type "solve"
Select solve[Equation]
Type in equation, press enter

B. $E(50) = .5$ demand is inelastic, raise price → increase in revenues

C. $E(100) = 2$ demand is elastic, raise price → decrease in revenues

Review Example 26: A biologist wants to study the growth of a certain strain of bacteria. She starts with a culture containing 25,000 bacteria. After three hours, the number of bacteria has grown to 63,000. ② How many bacteria will be present in the culture 6 hours after she started her study? ③ What will be the rate of growth 6 hours after she started her study? Assume the population grows exponentially and the growth is uninhibited.

Exponential models $\quad (0, 25000)$
$\quad (3, 63000)$

Spreadsheet view
enter points
create list
Fitexp[list1]

①

$$\boxed{f(x) = 25000e^{0.3081x}}$$

② EV $\quad f(6) = 158760$ $\quad \boxed{158760 \text{ bacteria}}$

③ ROC $\quad f'(6) = 48911.7811$ $\quad \boxed{\approx 48912 \text{ bacteria/hour}}$

(0, 50)   (17, 38.7)

Review Example 27: At the beginning of a study, there are 50 grams of a substance present. After 17 days, there are 38.7 grams remaining. How much of the substance will be present after 40 days? What will be the rate of decay on day 40 of the study? Assume the substance decays exponentially.

$f(x) = 50e^{-0.0151x}$

① $f(40) = 27.3643$ $\boxed{27.4 \text{ grams}}$

② $f'(40) =$ . . . . .

Halflife .... start with 300 mg. half life is 12 days

(0, 300)   (12, 150) ..... spreadsheet view / regression

Review Example ~~28~~ 29: Analyze the function: $f(x) = \frac{3}{2}x^4 - 2x^3 + 12x + 2$

from f

domain $(-\infty, \infty)$

zeros Root[polynomial]

asymptotes Asymptotes[function] none

from f'

graph f'

find zeros $\longrightarrow$ critical numbers

$x = -1$

number line   f' − − − − − | + + + + +

−1

relative min @ $x = -1$ rel min $\boxed{(-1, -6.5)}$

for y value find $f(-1) = -6.5$

f is incr. $(-1, \infty)$

f is decr $(-\infty, -1)$

from f''

graph f''

find zeros $\longrightarrow$ $f''(x) = 0$

Root [f'']

$x = 0$

$x = .6667$

number line

f'' : $+++$ $\cup$ | $- - -$ $\cap$ | $++++$ $\cup$

0 .6667

concavity intervals

f is conc ↓ on $(0, .6667)$

f is conc ↑ on $(-\infty, 0) \cup (.6667, \infty)$

inflection points

$(0, 2)$ $(.6667, 9.701)$

$f(0)$ $f(.6667)$

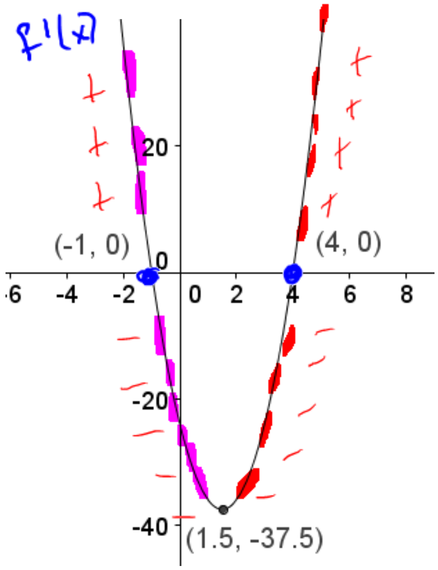
Review Example 28: Suppose a worker's production can be expressed using the function $N(t) = 65 - 18e^{-0.27t}$, where t gives the number of weeks since the worker started his/her job and $N(t)$ gives the number of units the worker can produce during her/his shift. At what rate would the worker's productivity be changing after 3 weeks on the job?

ROC

$N'(3) = 2.162$

$\approx$ 2units pershift/week

Review Example ~~29~~ 30: The graph given below is the *first derivative* of a function, *f*. Find the interval(s) on which the function is increasing, the interval(s) on which the function is decreasing, the *x* coordinate of each relative extremum (and state whether it is a relative maximum or a relative minimum), the interval(s) on which the function is concave upward, the interval(s) on which the function is concave downward and the *x* coordinate of any inflection points.



f' +++ | - - - - | +++++
-1, 4

f is inc. $(-\infty,-1) \cup (4,\infty)$

f is dec $(-1,4)$

f has a rel max at $x=-1$,

f has a rel min at $x=4$

f'' - - - - - - | +++++
1.5

f is conc ↓ on $(-\infty, 1.5)$

f is conc ↑ on $(1.5, \infty)$

inflection pt @ $x=1.5$

Review Example 31: Suppose your costs to produce your product can be express by $C(x)=0.001x^2 \quad 5x + 400$, where x is the number of items produced and $C(x)$ is the total cast to produce x items, given in dollars. If the demand for the product is modeled by the function $p=12 \quad 0.005x$, what is the maximum ~~revenue~~ profit?

① write rev. function $R(x)=xp = x(12-0.005x)$

enter $C(x)$ in GGB

② $P(x)=R(x)-C(x)$ $P(x) = R - C$

③ graph P' and find critical number

④ P(critical number)

Review Example 30: An apartment complex estimates that the revenues realized from rending out x of its 100 one-bedroom apartments can be modeled by the function $R(x) = -12x^2 + 2112x$. How many one bedroom apartments should be rented to maximize the revenue? What is the maximum revenue?

$n = 4$

Review Example 3~~1~~2: Use left endpoints and 4 subdivisions of the interval to approximate the area under $f(x) = 2x^2 + 1$ on the interval $[0, 2]$.

$\Delta x = \frac{b-a}{n}$ a b $\Delta x = \frac{2-0}{4} = \frac{1}{2}$

$[0, \frac{1}{2}][\frac{1}{2}, 1][1, \frac{3}{2}][\frac{3}{2}, 2]$

or

RectangleSum[f, start, end, # of rect, position]

RectangleSum[f, 0, 2, 4, 0] = 5.5

Review Example 3~~2~~3: Approximate the area between x axis and the graph of $f(x) = 0.3x^3 - 0.807x^2 - 3.252x + 6.717$ on [-2.82, 1.33] with

A. 50 rectangles and midpoints ~~endpoints~~.

RectangleSum [f, -2.82, 1.33, 50, .5] = 26.7597

B. 10 rectangles and right endpoints.

RectangleSum[f, -2.82, 1.33, 10, 1] = 26.3577

** **Review Example 3~~3~~4:** (Be able to do this by hand!!) Evaluate: $\int_1^3 (3x^2 + 4x - 7)dx$

① Find antiderivative

$$\int_1^3 (3x^2 + 4x - 7)\,dx$$

$$\frac{3x^3}{3} + \frac{4x^2}{2} - 7x \Big|_1^3 = x^3 + 2x^2 - 7x \Big|_1^3$$

② Evaluate

$$(3^3 + 2 \cdot 3^2 - 7 \cdot 3) - (1^3 + 2 \cdot 1^2) - 7 \cdot 1)$$

$24-(-4)=\boxed{28}$

Review Example ~~34~~ 35: Evaluate $\int_0^3 4x\left(x^2-3\right)^5 dx$

① Enter $f(x)=4x(x^2-3)^5$

② Integral[function, start, end]

Integral[f, 0, 3] = $\boxed{15309}$

** **Review Example ~~35~~ 36:** A study of worker productivity shows that the rate at which a typical worker can produce widgets on an assembly line can be expressed as $N(t)=-3t^2+12t+15$ where t gives the number of hours after a worker's shift has begun. ① Determine the number of widgets a worker can produce during the first hour of his/her shift. ② Determine the number of widgets a worker can produce during the last hour of a five hour shift.

[0,1] [1,2] [2,3] [3,4] [4,5]

first last

① $\int_0^1 (-3t^2+12t+15)\,dx = 20$ 20 units

Integral[N, 0,1]

② $\int_4^5 (-3t^2+12t+15)\,dt$ 8 units

Integral[N, 4, 5] = 8

** **Review Example ~~36~~ 37:** The median price of a house in a city in Arizona can be approximated by the function $f(t)=t^3-7t^2+17t+190$ where the median price is given in thousands of dollars and t is given as the number of years since 2000. This function has been shown to be valid for the years 2000 to 2005. Determine the average median price of a home in this city during this time period.

average value $\frac{1}{b-a}\int_a^b f(x)\,dx$ $[\overset{a}{0}, \overset{b}{5}]$

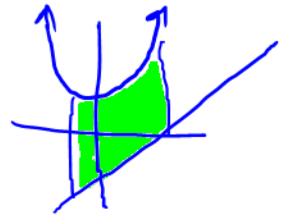
$\frac{1}{5-0}\int_0^5 (t^3-7t^2+17t+190)\,dt$

$\frac{1}{5}\int_0^5 (t^3-7t^2+17t+190)\,dt$

$\frac{1}{5}$ * Integral[f, 0, 5] = 205.4167

answer

\$205,417



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Review Example ~~37~~ 38: Find the area between the functions $f(x) = 3x^2 + 2$ and $g(x) = x - 3$ on the interval $-1 \le x \le 3$.

① graph

② set up definite integral

$$\int_{-1}^{3} \left[(3x^2+2) - (x-3)\right] dx$$

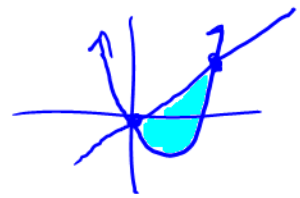
③ answer

$$\int_{-1}^{3} (3x^2 - x + 5)\, dx$$

Integral Between [f, g, -1, 3] = 44

**

Review Example ~~38~~ 39: Find the area of the region that is completely enclosed by the graphs of the functions $f(x) = x^2 - 3x$ and $g(x) = 1.6x$.



① graph

② find pts. of intersection

Intersect [f, g] $x = 0$ $x = 4.6$

③ $$\int_0^{4.6} \left[1.6x - (x^2 - 3x)\right] dx = \int_0^{4.6} (4.6x - x^2)\, dx$$

④ answer

Integral Between [g, f, 0, 4.6] = 16.2227

chapter 6

Review Example ~~39~~ 40: Suppose $f(x, y) = 3x^2y - 4xy + 6$. Compute $f(0,0)$, $f(2,-1)$ and $f(-1,-3)$.

$f(x, y) = 3x^2 y - 4x y + 6$

$f(0,0) = 6$

$f(2,-1) = 2$

$f(-1,-3) = -15$

Review Example ~~40~~ 41: Find the domain: $f(x, y) = \dfrac{x + 5y}{2x - y}$

$2x - y \neq 0$

$2x \neq y \longrightarrow y \neq 2x$

$\{(x, y) \mid y \neq 2x\}$

Review Example 42: Find the first partial derivatives of the function
$f(x,y) = 5x^2y^2 - 2x^3y + 9x^2 - 14y^2 + 10$.

Derivative [f, x] $\quad f_x = a(x,y) = -6x^2y + 10xy^2 + 18x$

Derivative [f, y] $\quad f_y = b(x,y) = -2x^3 + 10x^2y - 28y$

Review Example 43: Find the second-order partial derivatives of the function
$f(x,y) = 3x^2 - x^3y^3 + 5xy + 6y^3$.

Derivative [f, x] $\quad f_x = 6x - 3x^2y^3 + 5y \quad a(x,y)$

Derivative [f, y] $\quad f_y = -3x^3y^2 + 5x + 18y^2 \quad b(x,y)$

Derivative [a, x] $\quad f_{xx} = 6 - 6y^3$

Derivative [a, y] $\quad f_{xy} = -9x^2y^2 + 5$

Derivative [b, y] $\quad f_{yy} = -6x^3y + 36y$

Derivative [b, x] $\rightarrow f_{yx} = 6 - 6y^2$

Review Example 44: Evaluate the first and second-order partial derivatives of
$f(x,y) = 3x^2 - x^3y^3 + 5xy + 6y^3$ at the point (1, 2).

$f_x|_{(1,2)} = -11 \qquad a(1,2) =$

$f_y|_{(1,2)} = 65 \qquad b(1,2)$

$f_{xx}|_{(1,2)} = -42 \qquad c(1,2)$

$f_{xy}|_{(1,2)} = -31 \qquad d(1,2)$

$f_{yy}|_{(1,2)} = 60 \qquad e(1,2)$

$f_{yx}|_{(1,2)} = -31 \qquad g(1,2)$

Review Example ~~44~~ 45: Find the relative extrema of the function
$f(x,y)=x^2+2xy+2y^2-4x+8y-1.$

① $f_x = a(x,y) = 2x + 2y - 4$

$f_y = b(x,y) = 2x + 4y + 8$

② Find critical points

c: $2x+2y-4=0$

d: $2x+4y+8=0$

Intersect[c, d] = (8, -6)

③ Second order partials

→ f_{xx} = Deriv[a, x] = 2 > 0

f_{xy} = Deriv[a, y] = 2

f_{yy} = Deriv[b, y] = 4

f_{yx} = Deriv[b, x] = 2

④ Find $D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$D = 2 \cdot 4 - 2^2 = 8 - 4 = 4 > 0$

⑤ use 4 bullets

we have a relative min

⑥ rel. min $f(8,-6)$ = GGB

***

Review Example ~~43~~ 46: Find the relative extrema of the function $f(x,y)=x^3-2xy+y^2+5$.

① $f_x = 3x^2 - 2y$

$f_y = -2x + 2y$

② $3x^2 - 2y = 0$ c:

$-2x + 2y = 0$ d:

Intersect[c,d]

critical pts (0,0) $\left(\frac{2}{3}, \frac{2}{3}\right)$

③ 2nd order partials

⇒ $f_{xx} = 6x$ $f_{yy} = 2$

$f_{xy} = -2$ $f_{yx} = -2$

④ test (0,0) $f_{xx}\big|_{(0,0)} = 6(0) = 0$

$D(0,0) = 0 \cdot 2 - (-2)^2 = 0 - 4 = -4$

$D < 0$

saddle point

test $\left(\frac{2}{3}, \frac{2}{3}\right)$ $f_{xx}\big|_{\left(\frac{2}{3}, \frac{2}{3}\right)} = 6\left(\frac{2}{3}\right) = 4$

$D\left(\frac{2}{3}, \frac{2}{3}\right) = 4 \cdot 2 - (-2)^2$

$= 8 - 4 = 4 > 0$

$D > 0$ $f_{xx} > 0$

⑤ bullets

relative min

⑥ $f\left(\frac{2}{3}, \frac{2}{3}\right)$ = GGB