Sinking Fund

Is a fund accumulated over time in order to pay off a debt or meet future goals or obligations.

The periodic payment $E$ required to accumulate a sum of $F$ dollars over $n$ periods with interest charged at the rate of $i$ per period is

$$E = \frac{F i}{(1 + i)^n - 1}$$

Amortization

Is the process of paying off a debt with equal periodic payments made over a specified period of time that includes a portion of the principal and interest.

The periodic payment $E$ on a loan of $P$ dollars to be amortized over $n$ periods with interest charged at the rate of $i$ per period is

$$E = \frac{P i}{1 - (1 + i)^{-n}}$$

Example 1: Kelly wishes to buy a car that costs $32,998. The car dealer tells her that they can finance the car at 6.25% per year compounded monthly for 5 years. She decides to secure the loan from the dealer. How much will her monthly payments be?

$$i = \frac{0.0625}{12}$$

$$n = 60$$

$$\frac{i}{n} = \frac{0.0625}{60}$$

$$P = 32,998$$

$$E = \frac{32,998 \times 0.00525 \times 12}{1 - (1 + 0.00525/12)^{-60}}$$

$$E \approx 641.79$$
Example 2: A person would like to have $200,000 in an account for retirement 15 years from now. How much should be deposited quarterly in an account paying 6% per year compounded quarterly to obtain this amount?

\[ F = 200,000 \]
\[ t = 15 \]
\[ m = 4 \]
\[ r = 0.06 \]
\[ n = 60 \]
\[ i = \frac{0.06}{4} \]

\[ E = \frac{F \cdot i}{(1 + i)^n - 1} \]
\[ = \frac{200000 \times 0.06/4}{(1 + 0.06/4)^{60} - 1} \]
\[ = \$2,076.69 \]

Example 3: A sailboat costs $16,000. You pay 15% down and secure a loan for the remaining balance. How much are your monthly payments if 18% per year compounded monthly is charged over a period of 6 years?

Popper 2: Identify the type of problem.

a. Sinking Fund
b. Amortization

\[ t = 6 \times 12 \]
\[ r = 0.18 \]
\[ n = 12 \]
\[ P = 16000 - Dp \]
\[ 16000 - 0.15(16000) \]
\[ = 13600 \]

\[ E = \frac{P \cdot i}{1 - (1 + i)^{-n}} \]
\[ = \frac{13600 \times 0.18/12}{1 - (1 + 0.18/12)^{-72}} \]
\[ = \$310.19 \]

Example 4: Christina plans to go to Disney World in two summers and wishes to have $7000 by then. How much money should she deposit monthly for the next 2 years in an account paying 3.25% per year compounded monthly to achieve this goal?

\[ F = 7000 \]
\[ r = 0.0325 \]
\[ t = 2 \]
\[ m = 12 \]
\[ n = 24 \]
\[ i = \frac{0.0325}{12} \]

\[ E = \frac{F \cdot i}{(1 + i)^n - 1} \]
\[ = \frac{7000 \times 0.0325/12}{(1 + 0.0325/12)^{24} - 1} \]
\[ = \$282.68 \]
Example 5: Business partners, Bill and Bob, buy an apartment house for $1,250,000 by making a down payment of $125,000 and financing the rest with semiannual payments over the next 10 years. The interest rate on the debt is 8% per year compounded semiannually. How much is their semiannually payment?

\[ P = 1,250,000 - 125,000 = 1,125,000 \]

\[ t = 10 \]
\[ m = 2 \]
\[ r = 0.08 \]

\[ E = \frac{P \cdot i}{1 - (1+i)^{-m}} = \frac{1,125,000 \cdot 0.08/2}{1 - (1+0.08/2)^{-20}} = 82,778.47 \]

Popper 3: The Flores Family loves to go sailing on the weekends. Mr. Flores has decided to purchase a more spacious sailboat. The sailboat he is interested in buying in 3 years will cost him $20,000. An account at Invest Well Bank earns 2% per year compounded monthly. How much should Mr. Flores deposit in this account at the beginning of each month to be able to pay cash for the sailboat in 3 years?

a. 513.24  
b. 565.80  
c. 539.52  
d. 549.52  
e. 662.09
Chapter 4: Math of Finance Problems

Identify the type of problem.

1. Anna wants to have $5,000 saved when she graduates from college so that she will have a down payment for a new car. Her credit union pays 5% annual interest compounded monthly. How much money should she deposit each month to have the money available when she graduates in 3 years?

2. Bill bought a new car. His financing deal was a 5-year loan at 9.98% annual interest compounded monthly. His monthly payment was $421.25 and he paid no money down. What was the total purchase price of the car?

3. Sergio wants to have $5,000 in the bank in 3 years to pay for an Alaskan cruise. How much cash should he deposit today, if the bank pays 4% annual interest compounded quarterly, if he wants to be sure to have the funds available in 3 years?

4: Edwin and Frances are buying a new home. The purchase price is $155,000. They will make a 10% down payment on the house. Their loan for the house is a 30-year conventional loan at 6.75% per year compounded monthly. Find their monthly payment. Identify the type of problem.

5. Grace decides to start a savings program when she gets her first job after graduation. She deposits $2,500 into her credit union savings account. The credit union pays 3.8% annual interest compounded quarterly. How much money will she have in the account after 4 years?
Chapter 4: Math of Finance Problems

6. Helen bought a new computer. The finance company charged her 15% per year compounded monthly. Her monthly payments were $88.23 for 2 years and she made no down payment. What was the original price of the computer?

7. Gary decided to save some money for his daughter’s college education. He decided to save $300 per quarter. His credit union pays 4.5% per year compounded quarterly. How much money will he have available when his daughter starts college in 10 years?

8. Jolene owns a clothing store. She anticipates that she will need to replace her telephone system in 3 years. She projects that a new system will cost $12,500. Her bank pays 5% annual interest compounded semiannually. How much should she deposit semiannually in order to be able to pay cash for the new phone system?

9. Kris wins the lottery and decides to deposit $25,000 of his winnings in an account for his nephew. The bank pays 6.2% annual interest compounded monthly. How much will he be able to give his nephew in 5 years?

10. Megan bought a new car. Her car payments are $385.17 for 6 years. Her financing rate was 8.9% annual interest compounded monthly. She made a $1,200 down payment. What was the total purchase price of the car?
Chapter 4: Math of Finance Problems

11. A company has an immediate need for a loan. In an agreement worked out with its banker, the company assigns its royalty income of $4,800 per month for the next 3 years from certain oil properties to the bank, with the first payment due at the end of the first month. If the bank charges interest at the rate of 9% per year compounded monthly, what is the amount of the loan negotiated between the parties?

12. Carol’s employer deposits $1,000 per quarter into a retirement plan that earns 3.5% annual interest compounded quarterly. How much will be in the plan when she retires in 32 years?

13. Kelly wishes to buy a car that costs $32,998. The car dealer tells her that they can finance the car at 6.25% per year compounded monthly for 5 years. She decides to secure the loan from the dealer. How much will her monthly payments be?

14. David owns a small business and knows that he will need to purchase two new delivery vans in 5 years. He anticipates that the vans will cost the business $28,500 each. His bank pays 4.2% per year compounded monthly. How much should he deposit each month so that he will have the funds available to buy the vans in 5 years?

15. Mary deposited $5,000 in an account that earns 9% per year compounded monthly. How much will she have in 40 years, when she retires?

16. Denise wishes to have $6,000 in an account in 3 years. Her bank will pay 3.25% per year compounded semiannually. How much should she deposit now to have the desired amount of money in the account in 3 years?
Chapter 4: Math of Finance Problems

17. Parents agree to invest $500 at 10% per year compounded semi-annually for their son on the December 31 or June 30 following each semester that he makes the Dean’s list during his 4 years in college. If he makes the Dean’s list in each of the 8 semesters, how much money will his parents have to give him when he graduates in 4 years?

18. A health club offers to let you join for $50 down and payments of only $36 per month for 3 years. When you read the fine print, you discover that the interest rate is 18% per year compounded monthly. What is the cash price of the health club membership? How much will the club membership cost you after 3 years?

19. Nicholas and Olivia are buying a house for $250,000. They made a 15% down payment. Their financing is for 30 years at 6.78% annual interest compounded monthly. Find their monthly payment.

20. A lending company recently offered 36-month auto loans at 7.56% per year compounded monthly to applicants with good credit ratings. If you have a good credit rating and can afford monthly payments of $350, how much can you borrow from the company?
Chapter 4: Math of Finance Problems

Solutions
1. Sinking Fund; $129.02
2. Present Value of an Annuity; $19,835.47
3. Present Value with compound interest; $4,437.25
4. Amortization; $904.79
5. Future Value with compound interest; $2,908.31
6. Present Value of an Annuity; $2,084.79
7. Future Value of an Annuity; $15,050.05
8. Sinking Fund; $1,956.87
9. Future Value with compound interest; $34,058.44
10. Present Value of an Annuity; $22,626.97
11. Present Value of an Annuity; $150,944.67
12. Future Value of an Annuity; $234,281.12
13. Amortization; $641.79
14. Sinking Fund; $855.39
15. Future Value with compound interest; $180,549.51
16. Present Value with compound interest; $5,446.88
17. Future Value of an Annuity; $3,231.61
18. Present Value of an Annuity; $1,045.78; $1,346.00
19. Amortization; $1,382.51
20. Present Value of an Annuity; $11,241.81

Popper 5: A certain company has purchased new swivel chairs for its employees. The company
made the purchase on a credit plan at Buy Right. Their monthly payments are $1,000 for 3
years. Buy Right will charge 2.25% per year compounded monthly. How much was the original
total price of the furniture? Identify this type of problem.

a. Future Value with compound interest
b. Amortization
c. Future Value of an Annuity
d. Sinking Fund
e. Present Value of an Annuity
Section 5.1 – Sets and Set Operations

A collection of objects is called a set.

An object of a set is called an element.

Notation:
\( \in \) = “element of”
\( \notin \) = “not an element of”

Example 1: Let \( B = \{a, b, c, \ldots, y, z\} \). In set-builder notation, the set \( B \) can be written as follows:

\[
B = \{ x \in \text{the alphabet} \}
\]

Equality of Sets

Let \( A \) and \( B \) be two sets. We say that \( A \) is equal to \( B \), written as \( A = B \). This is true if and only if \( A \) and \( B \) have exactly the same elements. If two sets are not equal we write \( A \neq B \).

Subsets

Let \( A \) and \( B \) be two sets. We say that \( A \) is a subset of \( B \) or that \( A \) is contained in \( B \) and written \( A \subseteq B \). From the definition it follows that for any set \( A \), \( A \subseteq A \); that is, every set is a subset of itself.

Proper Subsets

If \( A \subseteq B \) but \( A \neq B \), then \( A \) is a proper subset of \( B \). If \( A \) is a proper subset of \( B \) then we write \( A \subset B \). In other words: \( A \) is a proper subset of \( B \) if the following two conditions hold.

1. \( A \subseteq B \)
2. There exist at least one element in \( B \) that is not in \( A \).

Example 2: Let \( A = \{1,2,3\} \), \( B = \{1,2,3,4,5\} \) and \( C = \{3,2,1\} \). In the following, answer true or false in the following:

\begin{align*}
A &= C & \text{T or F} \\
A &\subseteq C & \text{T or F} \\
A &\subset B & \text{T or F} \\
C &\subset A & \text{T or F} \\
5 &\notin C & \text{T or F}
\end{align*}

A set that contains no elements is called the Empty Set.

Note: We write \( \emptyset \) to denote the empty set. The symbol \( \emptyset \) is a subset of every set.
Example 3: Let \( A = \{a, b, c\} \). List all subsets and proper subsets of the set \( A \).

Subsets:  
\( \emptyset \), \( \{a\} \), \( \{b\} \), \( \{c\} \), \( \{a,b\} \), \( \{a,c\} \), \( \{b,c\} \), \( \{a,b,c\} \)

Proper Subsets (strictly smaller):
\( \emptyset \), \( \{a\} \), \( \{b\} \), \( \{c\} \), \( \{a,b\} \), \( \{a,c\} \), \( \{b,c\} \), \( \{a,b,c\} \)

The Universal set is the set of interest in a particular discussion.

A Venn diagram is a visual representation of sets.
They look like:

Set Operations

Let \( A \) and \( B \) be two sets. The set of all elements that that belong to either \( A \) or \( B \) or both is called the Union of \( A \) and \( B \) (denoted \( A \cup B \)).

In set builder notation \( A \cup B = \{x \mid x \in A \text{ or } x \in B \text{ or both}\} \)

Set Union in a Venn diagram looks like:
Let A and B be two sets. The set of all elements in common with both sets A and B is called the **Intersection** of A and B (denoted \( A \cap B \)).

In **set-builder notation** \( A \cap B = \{ x \mid x \in A \text{ and } x \in B \} \)

**Set Intersection in a Venn** diagram looks like:

If \( A \cap B = \emptyset \), then we say the intersection is the **null intersection** and that A and B are **disjoint**.

Let U be a universal set and \( A \subseteq U \). The set of all elements in U that are not in A is called the **Complement** of A. (denoted \( A^C \))

In **set-builder notation** \( A^C = \{ x \mid x \in U \text{, } x \notin A \} \)

**Set Complementation in a Venn diagram** looks like: