Test 3 Material

Math 1313 Section 4.1

**Section 4.1 – Simple Interest and Compound Interest: Future Value and Present Value**

**Simple Interest**

**Interest** is the amount of money paid for either borrowing money or earning money on a deposit.

**Simple Interest** is interest that is compounded on the original principal only.

$$I = Prt$$

**I** = Interest
**P** = principal (present value)
**r** = interest rate (% to decimal)
**t** = time in years

**Example 1:** Find the simple interest on a $1000 investment made for 3 years at an interest rate of 5% per year.

$I = Prt$

$= 1000(0.05)3$

$= \$150$

**Future Value with Simple Interest**

$$F = P(1 + rt) = \underbrace{P + Prt}_{I}$$

F = Future Value
P = Principal(present value)
r =interest rate
t = time in years

**Example 2:** Mike borrowed $1,200 at 10% simple interest per year. How much is due when the loan matures in 9 months?

$F = P(1+rt)$

$P = 1200$

$r = 0.10$

$t = \frac{9 \text{ months}}{12}$

12 months in a yr

$= 1200\left(1 + 0.10\left(\frac{9}{12}\right)\right)$

$= \boxed{\$1290}$

**Question 1:** Find the accumulated amount at the end of 14 months on a $250 bank deposit paying simple interest at a rate of 5% per year?

a. 251.36
b. 264.58
c. 305.32
d. 425.00

**Compounded Interest**

Interest that charged or earned on the original principal and also on any previously charged or earned interest.

**Future Value with Compound Interest Formula:**

Total number of times compounded

$$F = P(1+i)^n \qquad \text{where } i = \frac{r}{m} \text{ and } n = mt$$

**F** = Future Value

**P** = present value or principal.

**r** = the interest rate per year.

**m** = the number of compounding periods per year.

**t** = time in years.

$$\left.\begin{array}{ll} \text{Annually} & 1 \\ \text{Semi Annually} & 2 \\ \text{Quarterly} & 4 \\ \text{Monthly} & 12 \end{array}\right\} m$$

**Example 3:** Find the accumulated amount after 5 years if $1700 is invested at 6.25% per year compounded

$P = 1700 \quad t = 5 \quad r = 0.0625$

a. quarterly.

$m = 4$

$n = mt = 4(5) = 20$

$i = \frac{r}{m} = \frac{0.0625}{4}$

$F = P(1+i)^n$

$= 1700(1 + 0.0625/4)^{\wedge}20$

$= \boxed{\$2,318.02}$

b. semiannually.

$m = 2$

$n = mt = 2(5) = 10$

$i = \frac{r}{m} = \frac{0.0625}{2}$

$F = P(1+i)^n$

$= 1700(1 + 0.0625/2)^{\wedge}10$

$= \boxed{\$2,312.54}$

$$F = P(1+i)^n \qquad \frac{F}{(1+i)^n} = P$$

Math 1313  Section 4.1

**Present Value with Compound Interest Formula:**

$$P = F(1+i)^{-n} \qquad \text{where } i = \frac{r}{m} \text{ and } n = mt$$

**Example 4:** Kim and Ken find that they will need $15,500 to build an addition to their home in 4 years. How much should they invest now at 3.25% per year compounded quarterly to have the desired funds in 4 years?

P.V. w/ CI

$F = 15,500$
$t = 4\text{ yrs}$
$r = 0.0325$
$m = 4$
$n = mt = 4(4) = 16$
$i = \frac{r}{m} = \frac{0.0325}{4}$

$P = F(1+i)^{-n}$

$= 15500(1 + 0.0325/4)^{\wedge -16}$

$= \boxed{\$13,617.63}$

**Example 5:** A newborn child receives a $5000 gift towards a college education from her grandparents. How much will the $5000 be worth in 17 years if it is invested at 9% per year compounded quarterly?

F.V. w/ CI

$P = 5000$
$t = 17$
$r = 0.09$
$m = 4$
$n = mt = 68$
$i = \frac{r}{m} = \frac{0.09}{4}$

$F = P(1+i)^n$

$= 5000(1 + 0.09/4)^{\wedge 68}$

$= \boxed{\$22,702.60}$

**Example 6:** Kim invested a sum of money 4 years ago in a savings account that has since paid interest at the rate of 6.5% per year compounded monthly. Her investment is now worth $19,440.31. How much did she originally invest?

P.V. w/ CI

$F = 19440.31$
$t = 4$
$r = 0.065$
$m = 12$
$n = 48$
$i = \frac{0.065}{12}$

$P = F(1+i)^{-n}$

$= 19440.31(1 + 0.065/12)^{\wedge -48}$

$= \boxed{\$15000}$

**Question 4:** A grandmother deposited $8,000.00 in an account that pays 4% per year compounded annually when her granddaughter was born. What will the value of the account be when the granddaughter reaches her 18th birthday?

a. 8206.53
b. 16206.53
c. 34802.53
d. 205163.30

**Section 4.2 – Annuities: Future Value and Present Value**

An **ordinary annuity** is a sequence of equal periodic payments made at the end of each payment period.

Examples of annuities:

1. Regular deposits into a savings account.
2. Monthly home mortgage payments.
3. Payments into a retirement account.

We will study annuities that are subject to the following conditions:

1. The terms are given by fixed time intervals.
2. The periodic payments are equal in size.
3. The payments are made at the end of the payment periods.
4. The payment periods coincide with the interest conversion periods.

The **Future Value of an Annuity** is sum of all payments made and interest earned on an account.

**Future Value of an Annuity**

The future value $F$ of an annuity of $n$ payments of $E$ dollars each, paid at the end of each investment period into an account that earns interest at the rate of $i$ per period, is

$$F = E\left[\frac{(1+i)^n - 1}{i}\right]$$

F = Future value
E = Equal periodic payment
$i = \dfrac{r}{m}$
r = interest rate
m = compounding periods per year
n = $mt$
t = time in years

**Example 1:** Find the future value of the ordinary annuity of $1500 per semiannual period for 8 years at 9% per year compounded semiannually.

$E = 1500$
$t = 8$
$r = 0.09$
$m = 2$
$n = 16$
$i = \dfrac{0.09}{2}$

$$F = E\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 1500\left(\left(1 + 0.09/2\right)^{\wedge}16 - 1\right) / (0.09/2)$$

$$= \boxed{34,079.01}$$

**Present Value of an Annuity**

Why would we want to know the present value of an annuity?

Well in some cases we would like to know how much money we should put away now (at the present time), so that at a later time we could withdraw a certain amount of money for a certain length of time. In other cases we may wish to know how much something cost us originally.

The present value *P* of an annuity of n payments of *E* dollars each, paid at the end of each investment period into an account that earns interest at the rate of *i* per period, is

$$P = E\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$-n$

**Example 2:** Find the present value of the ordinary annuity of \$3000 per quarterly period for 6 years at 11% per year compounded quarterly.

$E = 3000$
$t = 6$
$r = 0.11$
$m = 4$
$n = 24$
$i = \frac{0.11}{4}$

$$P = E\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$= 3000\left(1-(1+0.11/4)^{\wedge -24}\right) / (0.11/4)$$

$$= \boxed{52{,}202.39}$$

**Example 3:** Carrie opened an IRA on January 31, 1990, with a contribution of \$2000. She made a contribution of \$2000 thereafter on January 31 of each year until her retirement in the year 2019 (30 payments). If the account earns interest at the rate of 6% per year compounded yearly, how much will Carrie have in her account when she retires?

F.V. A

$E = 2000$
$t = 30$
$m = 1$
$r = 0.06$
$n = 30$
$i = \frac{0.06}{1}$

$$F = E\left[\frac{(1+i)^{\wedge} - 1}{i}\right]$$

$$2000\left((1+0.06)^{30} - 1\right) / (0.06)$$

$$= \boxed{\$154{,}116.37}$$

**Example 4:** Parents of a college student wish to set up an account that will pay $350 per month to the student for 4 years. How much should they deposit now at 9% per year compounded monthly, so that their child will have the desired amount of money each month for 4 years?

P.V.A

$$P = E\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$= 350\left(1-(1+0.09/12)^{-48}\right)/(0.09/12)$$

$$= \boxed{14,064.67}$$

**Example 5:** Tim pays $320 per month for 4 years for a car, making a $2,000 payment. If the loan borrowed costs 6% per year compounded monthly, what was the original cost of the car?

$\hookrightarrow$ P.V.

$$P = E\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$= 320\left(1-(1+0.06/12)^{-48}\right)/(0.06/12)$$

$$= 13,625.70$$

$$+2,000.00$$

$$\boxed{\$15,625.70}$$

**Question 5:** Christopher's monthly net pay is 3,394.00. He decides to make monthly deposits of 10% of his monthly net pay into an account that pays 9.5% per year compounded monthly. How much will he have in the account after 2 years, assuming his net pay remains the same for the next two years?

a. 339.40
b. 8145.60
c. 7392.00
d. 8932.09
e. None of the above

**Example 6:** Tom and Jerri paid $10,000 down toward a new house. They also have a 30-year mortgage for which they pay $1,100 per month. If interest is 6.35% per year compounded monthly, what did the house that they purchased originally cost?

P.V.

$$P = E\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$= 1100\,(1-(1+0.0635/12)\wedge -360)/(0.0635/12)$$

$$= \$176{,}781.88 + 10{,}000$$

Down Payment

$$= \boxed{186{,}781.88}$$

$$1100 \times 30 \times 12 = 396{,}000 + 10K = 406{,}000$$

**Example 7:** Betsy estimates that she spends $150 per month on cigarettes. She decides that she will quit and invest the monthly amount spent on cigarettes in an account earning 2.25% per year compounded monthly. At the end of 5 years, she will donate the interest earned on her account to her favorite charity. How much money can the charity expect to get in 5 years?

F.V.

$$F = E\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 150\,((1+0.0225/12)\wedge 60 - 1)/(0.0225/12)$$

$$= 9{,}516.35$$

$$150 \times 5 \times 12 = 9000$$

$$\text{Interest} = 9{,}516.35 - 9000 = \boxed{516.35}$$