Practice Problems

1. Give the average value of \( f(x) = \cos(2x) \) on the interval \( \left[ \frac{-\pi}{2}, \frac{\pi}{4} \right] \).

2. The graph of \( f(x) \) is shown below.

   a. Give the area of the region bounded between the graph of \( f(x) \) and the x-axis on the interval \([-2, 3]\).

   b. \( \int_{-2}^{3} f(x) \, dx = \)

3. Give the average value of \( f(x) = x^2 - 2x + 4 \) on the interval \([-1, 2]\), and verify the conclusion of the mean value theorem for integrals for this function on this interval.

4. Sketch the region bounded between the graphs of \( f(x) = 3 - x^2 \) and \( g(x) = 2x \). Then find the area of the region.

5. Find the area bounded by the graph of \( f(x) = x^3 - x^2 \) and the x-axis on the interval \([0,2]\).

6. Sketch the region bounded by the curves \( x + y = 3 \) and \( x = y^2 + y \). Then give a formula for the area of the region involving integral(s) in \( x \). Repeat the process with integral(s) in \( y \). Finally, find the area of the region.
7. Sketch the region bounded between \( f(x) = 2x + 3 \) and \( g(x) = x^2 \). Rotate this region around the \( y \)-axis to generate a solid, and then find the volume of the solid.

8. Sketch the region in the first quadrant bounded between \( f(x) = 2x + 3 \) and \( g(x) = x^2 \). Rotate this region around the \( y \)-axis to generate a solid, and then find the volume of the solid.

9. Revolve the region bounded by the line \( y = 4 \) and the graph of \( f(x) = x^2 \) about the \( x \)-axis to generate a solid. Find the volume.

10. The region bounded between the graphs of \( f(x) = x^3 - x^2 \) and \( g(x) = 2x \) on the interval \([0,2]\) is rotated around the \( y \)-axis to generate a solid. Find the volume.

11. Sketch the region in the first quadrant bounded between the graphs of \( f(x) = x + 3 \) and 
    \[ g(x) = (x+1)^2. \]
    Rotate this region around the \( y \)-axis to generate a solid.
    
    a. Give a formula involving integral(s) in \( y \) for the volume generated.
    
    b. Give a formula involving integral(s) in \( x \) for the volume generated.
    
    c. Give the volume of the solid.

12. Repeat the previous problem, assuming that the region is rotated around the \( x \)-axis to generate a volume.

13. The base of a solid is given by the bounded between \( f(x) = 2x + 3 \) and \( g(x) = x^2 \). Cross sections taken perpendicular to the \( x \)-axis are squares. What is the volume of the solid.

14. The intersection of a solid with the \( xy \) plane is the region bounded between \( f(x) = 2x + 3 \) and 
    \( g(x) = x^2 \). Cross section taken perpendicular to the \( x \)-axis are circles whose diagonals are contained in the \( xy \) plane. What is the volume of the solid.