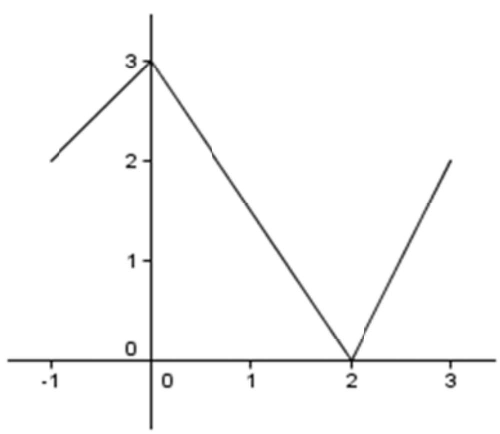
Practice Problems

1. Riemann sums. Sketch the region associated with the approximation, and then given the requested value.
   a. Give the upper sum for $f(x) = 4 - x^2$ on the interval [-1,1] with respect to the partition $P = \left\{-1, -\frac{1}{2}, \frac{1}{2}, 1\right\}$.
   b. Give the lower sum for $f(x) = 4 - x^2$ on the interval [-1,1] with respect to the partition $P = \left\{-1, -\frac{1}{2}, \frac{1}{2}, 1\right\}$.
   c. Give the midpoint approximation for $\int_{-1}^{1} (4 - x^2)\, dx$ with respect to the partition $P = \left\{-1, -\frac{1}{2}, \frac{1}{2}, 1\right\}$.
   d. Give the midpoint approximation for $\int_{-1}^{1} (4 - x^2)\, dx$ using $n = 4$.
   e. Give the left hand endpoint approximation for $\int_{-1}^{1} (4 - x^2)\, dx$ using $n = 4$.
   f. Give the right hand endpoint approximation for $\int_{-1}^{1} (4 - x^2)\, dx$ using $n = 4$.
   g. Use the graph of $f(x)$ below.



Give the upper sum for $f(x)$ on the interval [-1,3] with respect to the partition $P = \left\{-1, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, 3\right\}$.

2. Fundamental theorem of calculus.

   a. $\frac{d}{dx}\int_{\pi}^{x}\sin(t)\,dt =$

   b. $\frac{d}{dx}\int_{3x}^{\pi}\sin(t^2)\,dt =$

   c. $\frac{d}{dx}\int_{3x}^{2x}\sin(t^2)\,dt =$

   d. $\frac{d}{dx}\int_{1-2x}^{2x^3}\sin(t^2)\,dt =$

3. Basic Integration.

   a. $\int_{2}^{7} x\sqrt{x^2+2}\,dx =$

   b. $\int\left(3\sec^2(r)-2\sqrt{r-1}\right)dr =$

   c. $\int_{0}^{1}\frac{\cos(x)}{2+\sin(x)}\,dx =$

   d. $\int_{0}^{1} e^{-2x}\,dx =$

   e. $\int_{0}^{\pi/4}\frac{2}{1+x^2}\,dx =$

   f. $\int_{0}^{\pi/4}\sec(x)\tan(x)\,dx =$

   g. $\int\frac{3t}{t^4+1}\,dt =$

4. Area. Graph each region and find the requested area.

   a. Find the area bounded by the graph of $f(x)=1+x^2$ and the *x*-axis over the interval [-1,1].

   b. Find the area bounded by the graph of $f(x)=1-e^x$ and the *x*-axis over the interval [0,1].

   c. Find the area bounded by the graph of $f(x)=\sin(x)$ and the *x*-axis over the interval $[\pi/2,\pi]$.

   d. Give the area bounded between the *x*-axis and the graph of $f(x)=x^2+2x-3$ over the interval [-2,2].

5. Anti-derivatives.

a. Give the general anti-derivative for $g(x) = x^3 + 2x - \sqrt{x}$.

b. $F(x)$ is the anti-derivative for the function $x\sqrt{x^2+3}$ that satisfies $F(-1) = 2$. Give $3\left(F(0) - \sqrt{3}\right) + 1$.

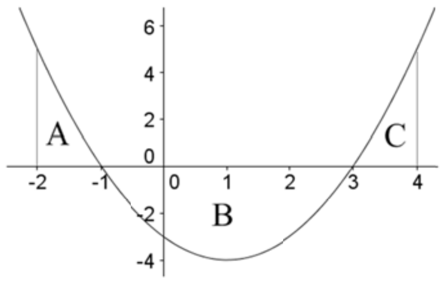
c. $F''(x) = x^2 - \frac{2}{\sqrt{x}} + 1$, $F'(1) = -3$ and $F(1) = 2$. Give $F(x)$.

d. Find a formula for $f(x)$, given that $2x^3 - 3x^2 + x - 1 = \int_{-1}^{x} f(t)dt$.

e. Suppose $f(x)$ is an anti-derivative of $r(x)$, and $g(x)$ is an anti-derivative of $s(x)$. We are given the data in the table about the functions f, g, r and s. $\int_1^3 \left(3r(x) - 2s(x)\right)dx =$

x	1	2	3	4
$f(x)$	3	2	1	4
$r(x)$	1	4	2	3
$g(x)$	2	1	4	3
$s(x)$	4	2	3	1

6. **Use the following information in parts a-c**. The graph of $f(x)$ is shown below, and $f(-2)=5$. The area of region A is 7/3, the area of region B is 34/3, and the area of region C is 7/3.



a. Give the area of the region bounded between the graph of $f(x)$ and the *x*-axis on the interval [-2 , 4].

b. $\int_{-1}^{4} f(x)dx =$

c. $\int_{-1}^{3/2} \left(3x - \frac{d}{dx} f(2x) \right) dx =$

d. The graph of $y = g'(x)$ is shown below, and $g(1)=1$. Give the values for $g(0)$, $g(2)$ and $g(3)$.