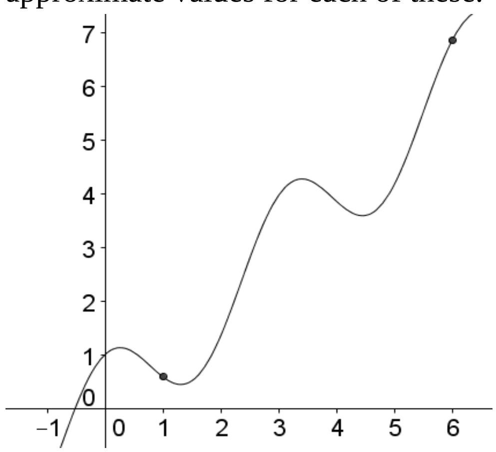
Practice Problems

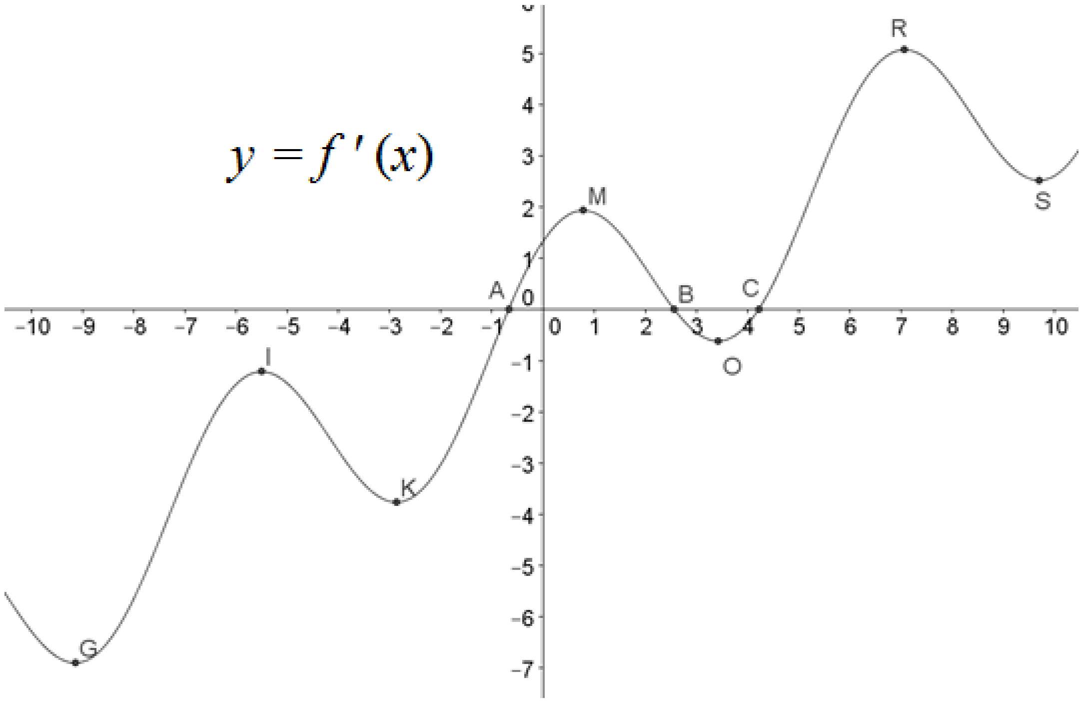
1. Differential Approximation (Tangent Line Approximation).
    a. Use the tangent line to $f(x) = \sin(x)$ at $x = 0$ to approximate $f(\pi/60)$.
    (Note: The phrase "use the tangent line" could be replaced with "use differentials." The phrase "at $x = 0$" could actually be omitted since $\pi/60$ is close to 0, and we know the function very well at 0.)
    b. Use differentials to approximate $(26)^{1/3}$.
2. The mean value theorem (for derivatives).
    a. State both the mean value theorem and Rolle's theorem.
    b. Explain how Rolle's theorem is a consequence of the mean value theorem.
    c. Verify the mean value theorem for the function $f(x) = x^3 - 4x^2 + x + 6$ on the interval $[-1, 2]$.
    d. Let $f(x)$ be a given differentiable function. Suppose $f(1) = 2$, and the value c that satisfies the mean value theorem on the interval $[1, 5]$ satisfies $f'(c) = 2$. Give the value for $f(5)$.
    e. Consider the graph of $f(x)$ below. Determine the number of values c that satisfy the conclusion of the mean value theorem on the interval [1,6], and give approximate values for each of these.



3. Find the critical numbers, the interval(s) of increase, and the interval(s) of decrease for the function $g(x) = -x^3 + 6x^2 + 15x - 2$.

4. Classifying critical numbers.
    a. What does it mean to say that $x = a$ is a critical number for a function $f(x)$?
    b. Describe the first derivative test for classifying a critical number for a function.
    c. Describe the second derivative test for classifying a critical number for a function.
    d. Give an example of a function and a critical value for which the second derivative test fails.
    e. Classify the critical number $x = 0$ for the function $f(x) = x^2 \cos(x)$.
    f. Find and classify all of the critical numbers for the function given in problem 3 using the first derivative test.
    g. Repeat the classification of critical numbers (if possible) from part f, using the second derivative test.
5. Absolute extrema (minimums and maximums).
    a. State the extreme value theorem.
    b. Explain the process of finding the extreme values of a continuous function $f(x)$ on an interval $\left[a,b\right]$.
    c. Find the absolute maximum and minimum values for the function $g(x) = -x^3 + 6x^2 + 15x - 2$ on the interval $\left[-1,2\right]$.
    d. Find the absolute minimum value of the function $f(x) = x^4 - 8x^2 + 3$.
6. Definitions.
    a. What does it mean to say that a function $f(x)$ is increasing on an interval?
    b. What does it mean to say that a function $f(x)$ is decreasing on an interval?
    c. What does it mean to say that a function $f(x)$ is concave up on an interval?
    d. What does it mean to say that a function $f(x)$ is concave down on an interval?
    e. What does it mean to say that a function $f(x)$ as inflection at a value $x = a$?
7. Concavity and inflection.
    a. Determine any intervals of concave up or concave down, and give any values where inflection occurs for the function $g(x) = -x^3 + 6x^2 + 15x - 2$.
    b. Determine any intervals of concave up or concave down, and give any values where inflection occurs for the function $f(x) = x^4 - 8x^2 + 3$.

8. The graph of $f'(x)$ is shown below on the interval [-11,11]. Give the intervals of increase, decrease, concave up, and concave down for $f(x)$. Also, find and classify any critical numbers for $f(x)$, and list any inflection for $f(x)$. Your answers should be given in terms of the labels given with the graph.



9. Determine the domain, any asymptotes, intervals of increase, intervals of decrease, intervals of concave up, intervals of concave down, critical numbers, and inflection for the function $f(x) = \dfrac{x-1}{x^2-5x+6}$. Then, graph the function.
10. What are the dimensions of the base of the rectangular box of greatest volume that can be constructed from 100 square inches of cardboard if the base is to be twice as long as it is wide? Assume the box has a top.
11. Find the point on the curve $y = x^2$ that is closest to the point $(2,3)$.
12. Give the dimensions of the rectangle with greatest area that has its base on the *x*-axis and its upper vertices on the parabola $y = 4 - x^2$.