Practice Problems

(Free response practice problems are indicated by "FR Practice")

1. Derivatives I.
   a. Define $f(x) = 3x - x^2$. Use the definition of derivative to give a formula for $f'(x)$.
   b. Define
   $$g(t) = \frac{1}{2t-1}$$
   Use the definition of derivative to give a formula for $g'(t)$.
   c. Let $f(x) = 3x^2 - 2x^3$. Give the value of $\lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$.
   d. Let $f(x) = \sin(2x)$. Give the value of $\lim_{x\to 1} \frac{f(x) - f(1)}{x-1}$.
2. Derivatives II. Give the derivative of each function.
   a. $f(x) = \tan(x^2+1) - 3x^5 - 2x + 1$
   b. $g(x) = \frac{-2x^2 - 8x - 1}{x-2}$
   c. $R(t) = \sin(2t) - \sqrt{3t-1} + \frac{1}{t^2}$
   d. $F(z) = \cos^3(2z-1)\sin(3z)$
   e. $G(x) = 5(\sin(2x-3) + 3x^3 - 2x)^4$
3. Derivatives III.
   a. Give the y-intercept of the tangent line to the graph of $f(x) = \sin(x) - \cos(x)$ at the point where $x = \pi$.
   b. Give the **largest** value of x for which the derivative of $f(x) = x^3 - 3x + 2$ is 0.
   c. $\frac{d}{dx}\left(\frac{\sin(x) + 2x}{\cos(x)}\right) =$
   d. $\frac{d^2}{dx^2}\left(-2x^3 + 5x^2 - 7\cos(\pi x)\right) =$
   e. Let $f(x) = -2x^3 + 5x^2 - 7\cos(\pi x)$. Give the normal line to the graph of $f''(x)$ at $x = 1/2$.
   f. $\frac{d^3}{dx^3}\left(\cos(x) - \sin(x)\right) =$
4. Chain rule.
   a. $G(x) = f(v(x))$, and each of f and v are differentiable functions. Suppose $f(1) = 2,\ f(3) = -1,\ f'(1) = -1,\ f'(3) = 5,\ v(2) = 3$ and $v'(2) = 4$. Give $G'(2)$.

b. $G(x) = f(v(x)) - 3v(x)$, and each of f and v are differentiable functions. Suppose $f(1) = 3$, $f(3) = -2$, $f'(1) = 1$, $f'(3) = 4$, $v(2) = 1$ and $v'(2) = 3$. Give $G'(2)$.

c. $f(x) = u(v(x)) + 3v(x)$, and each of u and v are differentiable functions. Also, $u(3) = -1$, $u(2) = 2$, $v(3) = 4$, $v(2) = 3$, $u'(3) = -2$, $u'(2) = 3$, $v'(3) = 5$ and $v'(2) = 4$. Find $f'(2)$.

d. $G(x) = \dfrac{(u(x)+v(x))^2 + 2x}{3v(x)-1}$, and each of u and v are differentiable functions. Also, $u(3) = -1$, $u(2) = 2$, $v(3) = 4$, $v(2) = 3$, $u'(3) = -2$, $u'(2) = 3$, $v'(3) = 5$ and $v'(2) = 4$. Find $G'(2)$.

5. Rates I.

a. Give the rate of change of the surface area of a sphere with respect to its radius when the radius is 2 units.

b. An object is moving along the graph of $f(x) = x^2$. When it reaches the point (2,4), the *x* coordinate of the object is decreasing at the rate of 3 units/sec. Give the rate of change of the distance between the object and the point (0,1) at the instant when the object is at (2,4).

c. A balloon retains a spherical shape as it is inflated. In addition, the balloon has a volume that is increasing at the constant rate of 1 cm^3/sec. Give the rate of change in the surface area of the balloon when r = 1 cm.

d. A right circular cone is expanding, and its height and radius are always equal. Also, the volume of the cone is increasing at the rate of 2 cubic inches per minute. How fast is the radius growing when the height is 2 inches?

6. Rates II.

a. The surface area of a sphere is increasing at the rate of 3 cm^2/sec. Give the rate of change of the volume of the sphere when the radius is 2 cm.

b. A heap of rubbish in the shape of a cube is being compacted. As it compacts, it retains its cubic shape. The change in the width of the cube is given by the function $\dfrac{dx}{dt} = -\dfrac{1}{t^2+1}$ in/sec, and $x = 4$ inches when $t = 2$ sec. Give the change in the volume of the cube when $t = 2$ sec.

c. A right circular cylinder is expanding, and its height and radius are always equal. Also, the volume of the cylinder is increasing at the rate of 1/ 2 cubic inches per minute. How fast is the surface area growing when the height of the cylinder is 2 inches?

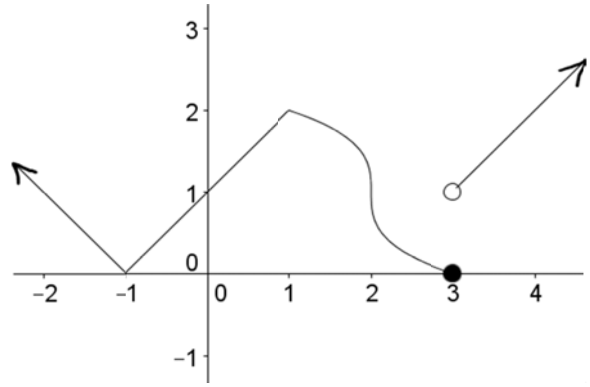
7. Rates III.

a. On a morning of a day when the sun will pass directly overhead, the shadow of a 40 ft building on level ground is 30 feet long. At the moment in question, the angle the sun makes with the ground is increasing at the rate of $\pi/1500$ radians per minute. At what rate is the shadow length decreasing at this instant?

b. Aman and Hisson are flying a kite. Aman is releasing the string at the constant rate of 10 ft/sec. At a praticular moment, Hisson is jogging along the ground directly below the kite at the rate of 6 ft/sec, at a distance of 300 ft from Aman. Aman had let out 500 ft prior to that moment. How fast is the kite rising at this moment?

c. The diameter and height of a right circular cylinder are found at a certain instant to be 10 cm and 20 cm respectively. If the diameter of the cylinder is increasing at the rate of 3 cm/sec, what change in height will keep the volume constant?

8. The graph of the function $f(x)$ is shown below.



a. Give the values where $f'(x)$ does not exit. Justify your answers.

b. Give an approximate sketch of the graph of $f'(x)$.

9. Determine whether each of following statements is **true or false**. Give reasons for your answers.

a. If $f(x)$ is a function and $\lim_{h\to 0}\dfrac{f(h)-f(-h)}{2h}=0$, then $f'(0)$ exists.

b. The function $f(x)=\dfrac{3}{4}(x-1)^{2/3}$ is differentiable at every value of x.

c. The definition of derivative can be used to show that $f(x)=|x|$ is not differentiable at $x=0$.

d. $f(x)=x^{1/3}$ is differentiable at $x=0$.

e. $f(x)=\begin{cases} ax^2-x, & x<1 \\ 2, & x=1 \\ bx+\dfrac{c}{x}, & x>1 \end{cases}$. It is possible to find constants a, b and c so that $f(x)$ is differentiable at $x=1$.

10. Derivatives and Inverses.

a. The function $f(x)=x^5+2x^3+3x+1$ is invertible. Give the y-intercept for the tangent line to the graph of $f^{-1}(x)$ at $x=1$.

b. Suppose $f(-1)=3$ and $f'(-1)=2$. Give the y-intercept of the tangent line to the graph of $f^{-1}(x)$ at $x=3$.

c. Suppose $\left(f^{-1}\right)'(1)=1/3$ and $f(3)=1$. Give the y-intercept of the normal line to the graph of $f(x)$ at $x = 3$.

11. (FR Practice) Let $s(t)=t^3-6t^2$ be the position of a particle in meters along an axis at time t minutes.
    a. Give expressions for the velocity and acceleration of the particle.
    b. Create a table showing the position, velocity and acceleration of the particle at times $t = 0, 1, 2, 3, 4, 5$ minutes. At each time, indicate whether the particle is speeding up or slowing down.
    c. Find all of the times when the particle is at rest.
    d. Give all of the values of t when the particle is moving forward.
    e. Give all of the values of t when the particle is moving backwards.
    f. Give all of the values of t when the particle is speeding up.
    g. Give all of the values of t when the particle is slowing down.
12. (FR Practice) Gravel is being poured by a conveyor onto a pile in such a way that it always retains a conical shape. In addition, the volume of the pile is increasing at the constant rate of 60π cubic feet per minute. Frictional forces cause the height of the pile to always be 1/3 of the radius.
    a. Create a detailed diagram illustrating the process described above.
    b. How fast is the radius of the pile changing at the instant when the radius is 5 feet?
    c. How fast is the height of the pile changing at the instant when the radius is 5 feet?
13. (FR Practice) Give complete answers to the following questions.
    a. Give the limit definition of the derivative of a function $f(x)$.
    b. Use the definition of derivative to give a formula for the derivative of $f(x)=\sqrt{2x+1}$.
    c. Use the formula found in part b to give an equation for the tangent line to the graph of $f(x)$ at $x=4$.
    d. Use the formula found in part b to find the y-intercept of the normal line to the graph of $f(x)$ at $x=4$.
14. (FR Practice) Give complete answers to the following questions.
    a. Verify that $(1,-1)$ is on the graph of $2x^2+xy+y^3=x+y$.
    b. Give a formula for dy/dx at each point on the graph of $2x^2+xy+y^3=x+y$.
    c. Find an equation for the tangent line to the graph of $2x^2+xy+y^3=x+y$ at the point $(1,-1)$.
    d. Give a formula for y'' at each point on the graph of $2x^2+xy+y^3=x+y$.