BC Practice Exam: Free Response, Part I. Graphing calculators may be used.

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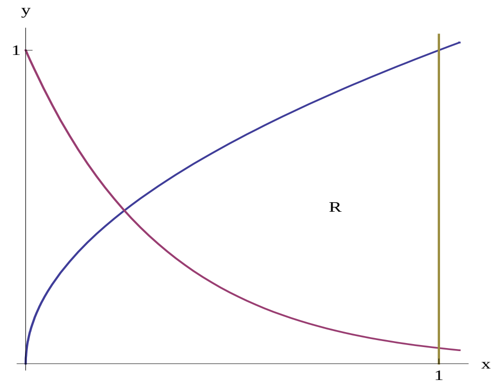
1. Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$, and the vertical line $x = 1$. See the figure.



   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in R. Find the volume of this solid.

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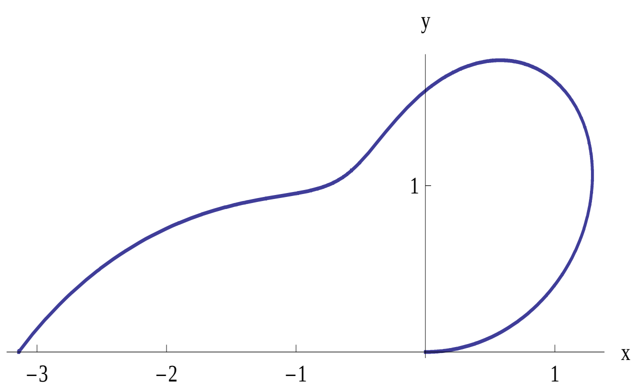
2. The curve shown below is the graph of the polar equation $r = \theta + \sin 2\theta$, for $0 \le \theta \le \pi$.



   (a) Find the area bounded by the curve and the x-axis.

   (b) Find the angle θ that corresponds to the point on the curve with x-coordinate -2.

   (c) For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this say about r? What does this say about the curve?

   (d) Find the value of θ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve with greatest distance from the origin. Justify your answer.

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3. The velocity of an object in motion in the plane for $0 \le t \le 1$ is given by the vector

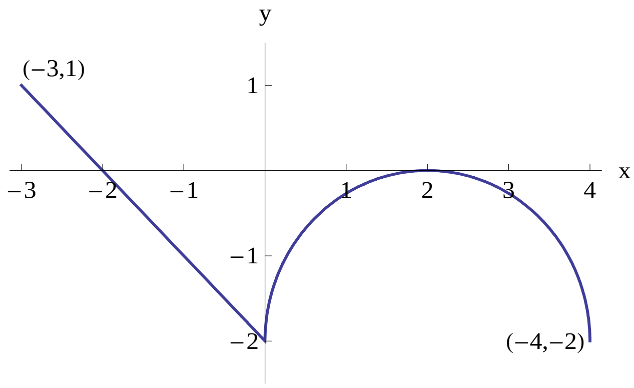
$$\mathbf{v}(t) = \left(\frac{1}{\sqrt{4-t^2}}, \frac{t}{\sqrt{4-t^2}} \right).$$

(a) When is the object at rest?

(b) If this object was at the origin when $t = 0$, what are its speed and position when $t = 1$?

(c) Find an equation of the curve the object follows, expressing y as a function of x.

4. Let f be a function defined on the closed interval $-3 \leq x \leq 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of a line segment and a semi-circle, as shown in the figure.



(a) On what intervals, if any, is f increasing? Justify your answer.

(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answer.

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5. Consider the differential equation

$$\frac{dy}{dx} = 5x^2 - \frac{6}{y-2}, \qquad y \neq 2.$$

Let $y = f(x)$ be the particular solution of this differential equation which satisfies the initial condition $f(-1) = -4$.

(a) Evaluate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $(-1, -4)$.

(b) Is it possible for the x-axis to be tangent to the graph of f at some point? Explain why or why not.

(c) Find the second degree Taylor polynomial for f about $x = 1$.

(d) Use Euler's method, starting at $x = -1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.

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6. The function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots$$

for all real numbers x.

(a) Find $f'(0)$ and $f''(0)$. Determine whether f has a local maximum, a local minimum, or neither at $x = 0$. Give a reason for your answer.

(b) Show that $1 - \frac{1}{3!}$ approximates $f(1)$ with error less than $\frac{1}{100}$.

(c) Show that $y = f(x)$ is a solution to the differential equation $xy' + y = \cos x$.