AB Practice Exam: Free Response, Part I. Graphing calculators may be used.

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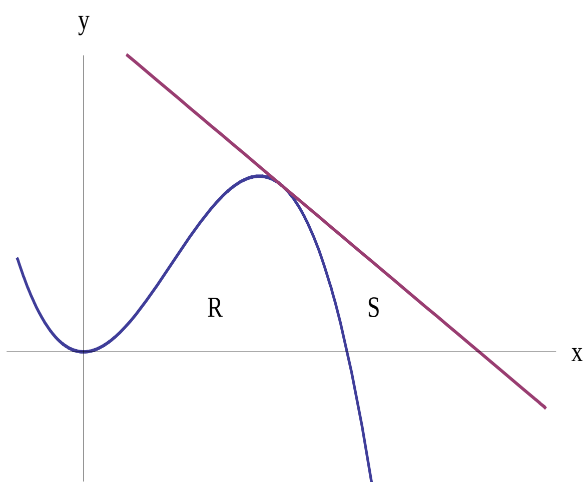
1. Set $f(x) = 4x^2 - x^3$, and let $\mathcal{L}$ be the line $y = 18 - 3x$, where $\mathcal{L}$ is tangent to the graph of f. Let S be the region bounded by the graph of f, the line $\mathcal{L}$ and the x-axis. The area of S is:



   (a) Show that $\mathcal{L}$ is tangent to the graph of f at the point $x = 3$.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when R is revolved about the x-axis.

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2. A tank contains 125 gallons of oil at time $t = 0$. During the time interval $0 \leq t \leq 12$, oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{[1 + \ln(t+1)]} \text{ gallons per hour.}$$

During the same time interval, oil is being removed from the tank at the rate

$$R(t) = 12 \sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

   (a) How many gallons of oil are being pumped into the tank during the time interval $0 \leq t \leq 12$?

   (b) Is the level of oil in the tank rising or falling at time $t = 6$ hours. Give a reason for your answer.

   (c) How many gallons of oil are in the tank at time $t = 12$ hours?

   (d) At what time t, for $0 \leq t \leq 12$, is the volume of oil in the tank the least? Justify your conclusion.

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3. A particle moves along the x-axis so that its velocity v at time t, for $0 \leq t \leq 5$, is given by

$$v(t) = \ln(t^2 - 3t + 3).$$

The particle is at the point $x = 8$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 4$.

(b) Find all the times in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 < t < 5$, does the particle travel to the left?

(c) Find the position of the particle at time $t = 2$.

(d) Find the average speed of the particle over the interval $0 \leq t \leq 2$.

AB Practice Exam: Free Response, Part II. Calculators may not be used.

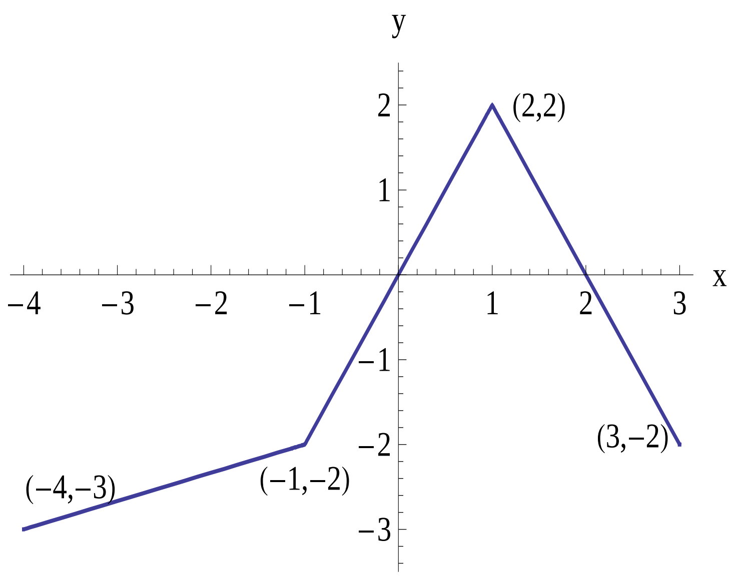
4. The graph of the function f consists of three line segments.

   (a) Let g be the function defined by $g(x) = \int_{-4}^{x} f(t)\,dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$ find the value of state that it does not exist.

   (b) For the function g given in part (a), find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

   (c) Let h be the function defined by $h(x) = \int_{x}^{3} f(t)\,dt$. Find all the values of x in the closed interval $-4 \le x \le 3$ for which $h(x) = 0$.

   (d) For the function h given in part (c), find all the intervals on which h is decreasing. Explain your reasoning.



5. Consider the curve given by $y^2 = 2 + xy$.

   (a) Show that $\dfrac{dy}{dx} = \dfrac{y}{2y - x}$.

   (b) Find all the points on the curve where the line tangent to the curve has slope $\dfrac{1}{2}$.

   (c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

   (d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $dy/dt = 6$. Find the value of dx/dt at time $t = 5$.

6. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1}, & 0 \leq x \leq 3 \\ 5 - x, & 3 < x \leq 5 \end{cases}.$$

(a) Is f continuous at $x = 3$? Explain why or why not.

(b) Find the average value of f on the closed interval $0 \leq x \leq 5$.

(c) Suppose that g is the function defined by

$$f(x) = \begin{cases} k\sqrt{x+1}, & 0 \leq x \leq 3 \\ mx + 2, & 3 < x \leq 5 \end{cases},$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m?