1. Set \( f(x) = 4x^2 - x^3 \), and let \( \mathcal{L} \) be the line \( y = 18 - 3x \), where \( \mathcal{L} \) is tangent to the graph of \( f \). Let \( S \) be the region bounded by the graph of \( f \), the line \( \mathcal{L} \) and the \( x \)-axis. The area of \( S \) is:

\[
\int_{x_1}^{x_2} (4x^2 - x^3) - (18 - 3x) \, dx
\]

(a) Show that \( \mathcal{L} \) is tangent to the graph of \( f \) at the point \( x = 3 \).

(b) Find the area of \( S \).

(c) Find the volume of the solid generated when \( R \) is revolved about the \( x \)-axis.

2. A tank contains 125 gallons of oil at time \( t = 0 \). During the time interval \( 0 \leq t \leq 12 \), oil is pumped into the tank at the rate

\[
H(t) = 2 + \frac{10}{1 + \ln(t + 1)} \text{ gallons per hour.}
\]

During the same time interval, oil is being removed from the tank at the rate

\[
R(t) = 12 \sin \left( \frac{t^2}{47} \right) \text{ gallons per hour.}
\]

(a) How many gallons of oil are being pumped into the tank during the time interval \( 0 \leq t \leq 12 \)?

(b) Is the level of oil in the tank rising or falling at time \( t = 6 \) hours. Give a reason for your answer.

(c) How many gallons of oil are in the tank at time \( t = 12 \) hours?

(d) At what time \( t \), for \( 0 \leq t \leq 12 \), is the volume of oil in the tank the least? Justify your conclusion.
3. A particle moves along the $x$-axis so that its velocity $v$ at time $t$, for $0 \leq t \leq 5$, is given by

$$v(t) = \ln(t^2 - 3t + 3).$$

The particle is at the point $x = 8$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 4$.

(b) Find all the times in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 < t < 5$, does the particle travel to the left?

(c) Find the position of the particle at time $t = 2$.

(d) Find the average speed of the particle over the interval $0 \leq t \leq 2$. 

4. The graph of the function \( f \) consists of three line segments.

(a) Let \( g \) be the function defined by \( g(x) = \int_{-4}^{x} f(t) \, dt \). For each of \( g(-1) \), \( g'(-1) \), and \( g''(-1) \) find the value of state that it does not exist.

(b) For the function \( g \) given in part (a), find the \( x \)-coordinate of each point of inflection of the graph of \( g \) on the open interval \(-4 < x < 3\). Explain your reasoning.

(c) Let \( h \) be the function defined by \( h(x) = \int_{x}^{3} f(t) \, dt \). Find all the values of \( x \) in the closed interval \(-4 \leq x \leq 3\) for which \( h(x) = 0 \).

(d) For the function \( h \) given in part (c), find all the intervals on which \( h \) is decreasing. Explain your reasoning.

5. Consider the curve given by \( y^2 = 2 + xy \).

(a) Show that \( \frac{dy}{dx} = \frac{y}{2y - x} \).

(b) Find all the points on the curve where the line tangent to the curve has slope \( \frac{1}{2} \).

(c) Show that there are no points \((x, y)\) on the curve where the line tangent to the curve is horizontal.

(d) Let \( x \) and \( y \) be functions of time \( t \) that are related by the equation \( y^2 = 2 + xy \). At time \( t = 5 \), the value of \( y \) is 3 and \( \frac{dy}{dt} = 6 \). Find the value of \( \frac{dx}{dt} \) at time \( t = 5 \).
6. Let $f$ be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1}, & 0 \leq x \leq 3 \\ 5 - x, & 3 < x \leq 5 \end{cases}.$$ 

(a) Is $f$ continuous at $x = 3$? Explain why or why not.

(b) Find the average value of $f$ on the closed interval $0 \leq x \leq 5$.

(c) Suppose that $g$ is the function defined by

$$f(x) = \begin{cases} k\sqrt{x+1}, & 0 \leq x \leq 3 \\ mx + 2, & 3 < x \leq 5 \end{cases},$$

where $k$ and $m$ are constants. If $g$ is differentiable at $x = 3$, what are the values of $k$ and $m$?