

Make a Better Problem: Change the Question

Purpose:

Participants will determine how to make a better problem by changing the attributes of the question.

Overview:

Participants will observe how changes in the wording of a problem can increase the rigor and make it a better problem. Participants will discuss what makes a better problem. Finally, participants will take a routine problem and work in groups to make a better problem.

TEXES Mathematics 4-8 Competencies. The beginning teacher:

- II.004.A Uses inductive reasoning to identify, extend, and create patterns using concrete models, figures, numbers, and algebraic expressions.
- III.008.E Applies the Pythagorean theorem, proportional reasoning, and right triangle trigonometry to solve problems.
- V. 015.C Demonstrates an understanding of the use of inductive reasoning to make conjectures and deductive methods to evaluate the validity of conjectures.
- V. 015.D Applies knowledge of the use of formal and informal reasoning to explore, investigate, and justify mathematical ideas.
- V.015.E Recognizes that a mathematical problem can be solved in a variety of ways and selects an appropriate strategy for a given problem.
- V.015.F Evaluates the reasonableness of a solution to a given problem.
- V.016.A Recognizes and uses multiple representations of a mathematical concept (e.g., a point and its coordinates, the area of circle as a quadratic function in r , probability as the ratio of two areas).
- V.016.C Expresses mathematical statements using developmentally appropriate language, standard English, mathematical language, and symbolic mathematics.
- V.016.D Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphic, pictorial, symbolic, concrete).
- V.016.F Uses the language of mathematics as a precise means of expressing mathematical ideas.
- V.017.A Applies theories and principles of learning mathematics to plan appropriate instructional activities for all students.
- VI.017.E Understands how to motivate students and actively engage them in the learning process by using a variety of interesting, challenging, and worthwhile mathematical tasks in individual, small-group, and large-group settings.
- VI.018.A Demonstrates an understanding of a variety of instructional methods, tools, and tasks that promote students' ability to do mathematics described in the TEKS.

TEKS Mathematics Objectives. The student is expected to:

- 6.2B Use addition and subtraction to solve problems involving fractions and decimals.
- 6.3C Use ratios to make predictions in proportional situations
- 6.11A, 7.13A, 8.14A
 - Identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics
- 6.11B, 7.13B, 8.14B
 - Use a problem-solving model that incorporates understanding the problem, making a plan,

- carrying out the plan, and evaluating the solution for reasonableness.
- 6.11C, 7.13C, 8.14C
Select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem.
- 6.11D, 7.13D, 8.14D
Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
- 6.12A, 7.14A, 8.15A
Communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models.
- 6.12B, 7.14B, 8.15B
Evaluate the effectiveness of different representations to communicate ideas.
- 6.13A, 7.15A, 8.16A
Make conjectures from patterns or sets of examples and nonexamples.
- 6.13B, 7.15B, 8.16B
Validate his/her conclusions using mathematical properties and relationships.
- 7.2F
Select and use appropriate operations to solve problems and justify the selections.
- 7.3B
Estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.
- 8.2A
Select and use appropriate operations to solve problems and justify the selections.
- 8.3B
Estimate and find solutions to application problems involving percents and proportional relationships such as similarity and rates.

Materials.

For each group of 3-4 participants:

- Blank Transparency
- Overhead pens
- Centimeter cubes or color tiles

Transparencies.

- Rollin' Along Problem A
- Rollin' Along Problem B
- What Makes A Good Problem?
- Simple Changes = Better Problems
- Make A Better Problem
- What a Dream!
- Students, Count Off!
- Vote, Vote, Vote!!
- Making Cookies

Activity Sheet(s).

- Rollin' Along Activity A
- Rollin' Along Activity B

- Students, Count Off!
- Vote,Vote,Vote!!
- Making Cookies
- Textbook Problems

Procedure:

Steps	Questions/Math Notes
1. Ask participants to solve Rollin' Along Problem A individually, and then share solutions with their group. Have groups use transparencies to share as many different strategies as possible. Discuss whether Problem A is an exercise or a problem.	Ask someone to read Problem A on the transparency. <i>How many different strategies were used to solve this problem? What were they?</i> <i>Why is it important that students see a variety of problem-solving strategies?</i> <i>How does this help develop better problem-solvers?</i> Solution: 16 motorcycles, 20 cars
2. Ask participants to solve Rollin' Along Problem B using the previous instructions.	<i>Why is communication among students and teachers important in the problem-solving process?</i> Communication allows students to use mathematical language and reasoning skills. Student discourse enables a variety of strategies to be discussed and helps develop better problem solving skills. <i>What is the difference between the two problems?</i> There are 36 vehicles in Problem A. In Problem B, the number of vehicles is not given. <i>Which is a better/richer problem? Why?</i> Problem B because it has multiple answers and multiple strategies.
3. Ask participants to discuss "what makes a good problem" with their group. Have each group share one or two ideas. Display What Makes a Good Problem transparency.	<i>When you read a problem, what makes you think "that's a good problem"?</i> <i>When we are searching for good problems, what should we look for?</i>

4. Discuss making changes to improve a problem. Use Simple Changes = Better Problems transparency.	<i>How can you create a better problem from an original one?</i> <i>What change was made in Rollin' Along Problem A that generated Rollin' Along Problem B?</i> A condition (36 vehicles) was dropped.
5. Provide more examples of changing problems using What a Dream! Transparency. Assign different groups to solve problems 2,3, and 4 below. Ask each group what change or changes were made to each problem. Use the following transparencies: 1. What a Dream! 2. Student, Count Off!! 3. Vote! Vote! Vote! 4. Making Cookies	As each transparency is displayed ask the following questions: <i>What changes have been made?</i> 1. Change the context from Rollin' Along to What a dream. 2. Change the numbers 3. Ask a different question/ Change a condition 4. Add a condition and ask a different question. <i>How do the changes make the problems more challenging?</i> The rigor of the problem is increased. There are multiple strategies and/or solutions. Be sure to guard against changing a problem for the sake of change. The problem may be different but not richer.
6. Ask participants to read Make a Better Problem on the transparency. Have participants work in pairs to work the problem then change it to make a better problem. Have participants share their revised problems.	<i>What changes did you make to your problem?</i> <i>How did that change the richness of the problem?</i>
7. Debrief participants on the objectives for Day 3 – Participants will learn questioning strategies for themselves and their students to use in the problem solving process. In addition, participants will determine how to make a better problem by changing the attributes of the question.	<i>Describe the importance of questions in the problem solving process.</i> <i>What are the characteristics of a good problem?</i> <i>What are the characteristics of a good question?</i> <i>Is there a relationship between the two?</i>
8. Assign homework. Give participants the Textbook Questions activity sheet. Participants will solve the four problems then use one of the suggested methods to make each a better problem.	