Questioning in Problem Solving

Purpose:
Participants will learn questioning strategies for themselves and their students to use in the problem-solving process.

Overview:
Participants will share questions generated while working a problem. They will compile a list of general questions that could be asked when working most problems. Participants will also recognize the importance of developing heuristics for their students and see how the Problem-Solving Board contributes to this development.

TExES Mathematics 4-8 Competencies.

I.00.E Selects appropriate representations of real numbers (e.g., fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.
III.008.E Applies the Pythagorean theorem, proportional reasoning, and right triangle trigonometry to solve problems.
III.010.A Uses and understands the development of formulas to find lengths, perimeters, areas, and volumes of basic geometric figures.
V.015.D Applies knowledge of the use of formal and informal reasoning to explore, investigate, and justify mathematical ideas.
V.015.E Recognizes that a mathematical problem can be solved in a variety of ways and selects an appropriate strategy for a given problem.
V.015.F Evaluates the reasonableness of a solution to a given problem.
V.016.A Recognizes and uses multiple representations of a mathematical concept (e.g., a point and its coordinates, the area of circle as a quadratic function in r, probability as the ratio of two areas).
V.016.C Expresses mathematical statements using developmentally appropriate language, standard English, mathematical language, and symbolic mathematics.
V.016.D Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphic, pictorial, symbolic, concrete).
V.016.F Uses the language of mathematics as a precise means of expressing mathematical ideas.
VI.017.E Understands how to motivate students and actively engage them in the learning process by using a variety of interesting, challenging, and worthwhile mathematical tasks in individual, small-group, and large-group settings.
VI.017.G Recognizes the implications of current trends and research in mathematics and mathematics education.
VI.018.G Demonstrates an understanding of a variety of questioning strategies to encourage mathematical discourse and to help students analyze and evaluate their mathematical thinking.

TEKS Mathematics Objectives. The student is expected to:

6.1B generate equivalent forms of rational numbers including whole numbers, fractions, and decimals.
6.3A use ratios to describe proportional situations.
7.1B convert between fractions, decimals, whole numbers, and percents mentally, on paper, or with a calculator.
7.3B estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.
8.1B select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships.
8.3B estimate and find solutions to application problems involving percents and proportional relationships such as similarity and rates.
6.11A, 7.13A, 8.14A

identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics

6.11B, 7.13B, 8.14B
use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness.

6.11C, 7.13C, 8.14C
select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem.

6.11D, 7.13D, 8.14D
select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.

6.12A, 7.14A, 8.15A
communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models.

6.12B, 7.14B, 8.15B
evaluate the effectiveness of different representations to communicate ideas.

6.13A, 7.15A, 8.16A
make conjectures from patterns or sets of examples and nonexamples.

6.13B, 7.15B, 8.16B
validate his/her conclusions using mathematical properties and relationships.

Terms.
Percents, equivalent fractions, proportions, ratios.

Materials.
Problem-Solving Board poster
Popsicle sticks – one per participant

Transparencies.
- Top Twelve Questions
- What are Heuristics?
- Problem-Solving Board with Heuristics
- Tick Tock Problem
- Tick Tock Question Board
- High Flyin' Problem

Activity Sheet(s).
- High Flyin' Problem-Solving Board
- High Flyin' Question Board

Procedure:

Steps	Questions/Math Notes
1. Direct participants to write their name on a popsicle stick. Collect these.	
2. Share questions at each table that were generated for Euclid's Problem as part of the homework assignment. Each group should compile a list of at least ten questions and should address each stage of the Problem-	

Solving Board.	
3. Discuss questions compiled by groups. Select a popsicle stick and have that person share a question generated by his group. Write these questions on the Problem-Solving Board poster in the appropriate section. Continue selecting names from popsicle sticks until participants have no more questions to share	Point out that the popsicle sticks are a method that can be used to include more students in the discussion. They also ensure that all students are paying attention and are ready to participate since they do not know when their name will be called.
4. Analyze the questions on the poster board to see which questions are general and can be used in any problem and which are unique to Euclid's Problem. Make a mark next to the questions that are specific to Euclid's Problem.	*Which questions are specific to Euclid's Problem and which could be used for any problem-solving situation?*
5. Ask participants to rewrite the marked questions into general questions or prompts that could be used in any problem-solving situation. Generate a list of these questions or prompts.	*How can you rewrite the specific questions so that they could be used for any problem-solving situation?*
6. Share with participants the Top Twelve Questions transparency. These are questions they can use in any problem-solving situation. They can add to this list from the questions and prompts the class generated above.	Some teachers keep their Top Twelve List taped to their podiums or as posters in the back of the room. This becomes a quick reference for them when working with student groups.
7. Display "What are Heuristics?" transparency.	*Does everyone have the same set of heuristics?* No – as students work through problems, they incorporate new heuristics into their "bank" of heuristics. Depending on the type of problems students are presented with, they will choose only some of their heuristics to help them with the process.
8. Share a sample of Problem-Solving Heuristics embedded within the Problem-Solving Board.	These are a set of questions that students should ask themselves through the various stages of the problem-solving process. These do not include all possible questions, and not all are appropriate for every problem. The goal is for students to learn to ask these questions of themselves, thus becoming more efficient and independent problem solvers. At the beginning of problem solving, some teachers provide this board to students as a reference sheet. With continued use, the questions become more automatic and the students refer less to this board.
9. Display the Tick Tock Problem from Day 2. Give the Tick Tock Question Board to a volunteer. As the instructor is working the problem on the overhead projector, talk through all the steps of the solution process.	This activity models what the participants will be expected to do in the next step. It will help participants focus on the thought processes involved with problem solving.

The volunteer will record everything that is happening on the Tick Tock Question Board. If at any time the instructor becomes quiet, the volunteer should ask questions, such as "What are you thinking?" After several minutes of this, display what was recorded on the Tick Tock Question Board.	The Techniques section is to record specific behaviors that are occurring in the problem-solving process. For example, is the instructor • rewriting information? • highlighting information? • drawing pictures? • using directional arrows?
10. Display the High Flyin' Problem. Read the question aloud. Distribute the High Flyin' Problem-Solving Board and a High Flyin' Question Board to each pair of participants. One person will work the problem and verbalize their thought processes. The other person will record these thoughts on the Question Board, prompting with questions anytime there is silence.	This activity will help participants focus on the techniques and questions they ask themselves through the problem-solving process.
11. Debrief by asking participants to share the comments recorded on the Question Board.	*What questions did you ask yourself while working the problem?* Van De Walle suggests the three questions students should always ask themselves while solving a problem include: • What am I doing? • Why am I doing it? • How does it help me? *What other techniques were recorded?* *Did everyone process the problem the same way? If not, describe the differences.* *Were there strategies or heuristics that others used that would be helpful to you in the future? If so, describe them?* *Was it difficult to verbalize your thought processes? If so, why?* Teachers must be aware of their own heuristics in order to help students develop theirs.