Implementing Problem Solving

Purpose:
Participants will learn tools for implementing problem solving and will see how Bloom's Taxonomy is addressed.

Overview:
Participants will work a problem using the Problem-Solving Board. Participants will also listen for teacher questions throughout the process to recognize the importance of teacher questioning. Finally, participants will look at how Bloom's Taxonomy is addressed by each stage of the problem-solving model.

TExES Mathematics 4-8 Competencies. The beginning teacher:

III.008.A	Selects and uses appropriate units of measurement (e.g., temperature, money, mass, weight, area, capacity, density, percents, speed, acceleration) to quantify, compare, and communicate information.
III.008.B	Develops, justifies, and uses conversions within measurement systems.
III.008.E	Applies the Pythagorean theorem, proportional reasoning, and right triangle trigonometry to solve problems.
III.010.A	Uses and understands the development of formulas to find lengths, perimeters, areas, and volumes of basic geometric figures.
V.015.D	Applies knowledge of the use of formal and informal reasoning to explore, investigate, and justify mathematical ideas.
V.015.E	Recognizes that a mathematical problem can be solved in a variety of ways and selects an appropriate strategy for a given problem.
V.015.F	Evaluates the reasonableness of a solution to a given problem.
V.016.C	Expresses mathematical statements using developmentally appropriate language, standard English, mathematical language, and symbolic mathematics.
V.016.D	Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphic, pictorial, symbolic, concrete).
V.016.E	Demonstrates an understanding of the use of visual media such as graphs, tables, diagrams, and animations to communicate mathematical information.
V.016.F	Uses the language of mathematics as a precise means of expressing mathematical ideas.
V.016.G	Understands the structural properties common to the mathematical disciplines.
VI.017.A	Applies theories and principles of learning mathematics to plan appropriate instructional activities for all students.
VI.017.D	Understands how learning may be assisted through the use of mathematics manipulatives and technological tools.
VI.017.E	Understands how to motivate students and actively engage them in the learning process by using a variety of interesting, challenging, and worthwhile mathematical tasks in individual, small-group, and large-group settings.
VI.017.G	Recognizes the implications of current trends and research in mathematics and mathematics education.

TEKS Mathematics Objectives. The student is expected to:

6.6C	Describe the relationship between radius, diameter, and circumference of a circle.
6.8B	Select and use appropriate units, tools, or formulas to measure and to solve problems involving length (including perimeter and circumference), area, time, temperature, capacity, and weight.

6.11A	Identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics
6.11B	Use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness.
6.11C	Select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem.
6.11D	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
6.12A	Communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models.
6.12B	Evaluate the effectiveness of different representations to communicate ideas.
6.13A	Make conjectures from patterns or sets of examples and nonexamples.
6.13B	Validate his/her conclusions using mathematical properties and relationships.
7.2B	Use addition, subtraction, multiplication, and division to solve problems involving fractions and decimals.
7.9	Estimate measurements and solve application problems involving length (including perimeter and circumference), area, and volume.
7.13A	Identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics
7.13B	Use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness.
7.13C	Select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem.
7.13D	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
7.14A	Communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models.
7.14B	Evaluate the effectiveness of different representations to communicate ideas.
7.15A	Make conjectures from patterns or sets of examples and nonexamples.
7.15B	Validate his/her conclusions using mathematical properties and relationships.
8.1B	Select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships.
8.2A	Select and use appropriate operations to solve problems and justify the selections.
8.14A	Identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics
8.14B	Use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness.
8.14C	Select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem.
8.14D	Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
8.15A	Communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models.
8.15B	Evaluate the effectiveness of different representations to communicate ideas.
8.16A	Make conjectures from patterns or sets of examples and nonexamples.
8.16B	Validate his/her conclusions using mathematical properties and relationships.

Terms.

Radius, diameter, circumference, difference, dimensions, area, square units

Materials.

Problem-Solving Board – poster size

For each group of 3-4 participants:

- Post-it notes
- Overhead pens
- Problem-Solving Board transparency (2 transparencies)

Transparencies.

- Four-stage Problem-Solving Model
- Problem-Solving Board
- Tick Tock Problem
- Euclid's Tile and Carpet Company Problem
- Problem Solving Process
- Bloom's Taxonomy

Activity Sheet(s).

- Tick Tock Problem-Solving Board
- Euclid's Tile and Carpet Company Problem-Solving Board
- Euclid's Tile and Carpet Company Question Board

Steps	Questions/Math Notes
1. Review the Four-stage Problem-Solving Model (transparency). Ask participants to describe what the student is doing at each stage.	Possible answers: Stage 1 – Understand the Problem • Identify the question • Identify the knowns • Identify the unknowns • Draw a picture or sketch to help understand the problem and sort out information Stage 2 – Devise a Plan • Relate problem to previously worked problems to identify similarities and differences • Think about strategies used in the past on problems of this type • Decide on a strategy Stage 3 – Carry Out the Plan • Work through the problem • Evaluate effectiveness of plan – does the strategy need to change? Stage 4 – Review the Results • Is the answer reasonable? • Was there a more efficient way to solve the problem?

	• Did I answer the question asked? • Can I solve the problem another way? • Can I think of a way to extend the problem?
2. Show the Problem-Solving Board (Transparency) and how the four stages are represented on the board.	The Problem-Solving Board gives students a way to organize their thoughts. This can give them some guidance when they are struggling with what to do.
3. Pass out the Tick Tock Problem-Solving Board (Activity Sheet). Display the Tick Tock Problem on the overhead projector. In groups of 3-4, participants will work through the problem using the board. While participants are working, ask questions of the groups. When they are asked questions, participants will record the question on a post-it note for later in the lesson.	Possible questions for teacher to ask student groups about the problem: <u>Understand the Problem</u> *What are the facts?* *What is the question?* *Describe the problem.* *Are you sure you can assume…?* *How does the minute hand compare to the hour hand?* <u>Devise a Plan</u> *Have you worked a problem like this before? If so, what strategy did you use?* *Are there strategies you can eliminate? If so, which ones and why?* *What prior knowledge could you use to help solve this problem?* *Have you tried organizing the data or drawing a picture? If so, why did you decide to do this?* <u>Carry Out the Plan</u> *Describe what you are doing.* *How did you know to do this?* *Compare the movement of the hour hand to the minute hand.* *Describe the relationship between the hands and the face of the clock.* *Is your plan working? If so, how do you know? If not, what else might you try?* *Have you tried breaking the problem down into smaller problems? Would that help? Why or why not?* <u>Review the Results</u> *Analyze your answer – is it reasonable?* *Can you use your results to predict the difference in a 24-hour day?* *Evaluate your solution strategy. Would you use the same strategy the next time you are presented a problem like this? Why or why not?* *Is there another way to solve it? If so, how?* *Describe some changes you could add to the*

	problem to extend it.
4. Direct groups to write their solution on a Problem-Solving Board (transparency). Allow each group to share their solution. Use the debriefing questions on the right.	<u>Understand the Problem</u> *What types of information did you put in this section?* <u>Devise a Plan</u> *Did you select more than one strategy when you were initially presented with the problem? If so, which strategies did you select?* *Were there some strategies listed here that you knew would not be helpful? If so, which ones and why?* *Was there a strategy you could add here to solve this problem?* *Describe why the strategies are listed on the board.* <u>Carry Out the Plan</u> *Why is this section larger than any other section?* *Did you refer back to other sections while working in this section? If so, which ones and why did you refer back?* <u>Review the Results</u> *Share what you wrote here.* *Did you justify your answer?* *Did anyone estimate to confirm their results? If so, share that strategy.* *How did the other sections of the board aid while reviewing the results?*
5. Display Problem-Solving Process transparency.	Discuss the fact that problem solving is not a linear process. It is a dynamic process – you move in and out of the four stages of the problem solving model. When participants moved in and out of stages on the Problem-Solving Board to refer to previous or future stages, this was representative of the dynamic process.
6. Discuss the questions the participants recorded on their post-it notes. Ask participants to share at what stage of the Problem-Solving Board these questions were asked. As participants share their questions, have them place the post-it note on the Problem-Solving Board poster in the appropriate stage.	The questions asked by the teacher should lead participants to clarify their thoughts on particular aspects of the problem. The questions can also redirect students. Care must be taken however, not to lead the students too much toward the solution. An important component of problem solving is the struggle to figure out what to do when you don't know what to do.
7. Display Bloom's Taxonomy transparency.	*How does the Problem-Solving Board address*

Ask participants to compare Bloom's Taxonomy with the Problem-Solving Board. They may also want to refer to the questions generated on the Problem-Solving Board poster.	*Bloom's taxonomy?* *Describe where each of Bloom's levels is addressed on the Problem-Solving Board.* Possible answers: <u>Understand the Problem</u> • Knowledge (What information am I given?) • Comprehension (Describe the question being asked. Organize the information.) <u>Devise a Plan</u> • Knowledge (Have I worked a problem like this before?) • Comprehension (Could I draw a picture to represent the information? Select a strategy.) <u>Carry out the Plan</u> • Application (Applying information to produce a result.) • Analysis (Is my strategy working?) <u>Review the Results</u> • Analysis (Is my solution reasonable? Have I identified all possible answers?) • Synthesis (Could I extend this problem in some way?) • Evaluation (Determine whether a different strategy would have been more effective.)
8. Distribute the Euclid's Tile and Carpet Company Problem-Solving Board and a Problem-Solving Board transparency to participants. Read the problem aloud from Euclid's Tile and Carpet Company transparency. Allow five minutes of productive silence before allowing pairs to work together.	
9. Ask groups to share their solutions on the Problem-Solving Board transparencies. Try to have several strategies shared.	*What question was asked?* *Explain the relationship between the tile dimensions and the room dimensions.* *Would drawing a picture help? If so, why?* *What strategies were used by groups?* *Are there assumptions you made when solving this problem?* (tiles in horizontal rows versus tiles on the diagonal) *What difficulties did you encounter with this problem?* *Was there more than one possible solution?*

	How do you know? *What ideas can you add to extend this problem?*
10. Discuss the objective for the lesson – Participants will learn tools for implementing problem solving and will see how Bloom's Taxonomy is addressed.	*How was the objective addressed?* *Can you describe how Bloom's Taxonomy fits into the Problem-Solving Board?* *Describe some benefits to the use of the Problem-Solving Board.*
11. Distribute the Euclid's Tile and Carpet Company Question Board to participants. For homework, participants should write possible questions they could ask students at each stage.	