

Definition Set D

Definition I

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Problem Solving

Problem solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking.

By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. In everyday life and in the workplace, being a good problem solver can lead to great advantages.

Problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program. Problem solving in mathematics should involve all the five content areas described in these Standards. The contexts of the problems can vary from familiar experiences involving students' lives or the school day to applications involving the sciences or the world of work. Good problems will integrate multiple topics and will involve significant mathematics.

Definition II

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Problem Solving Standard for Grades 6 – 8

Through problem solving, students can experience the power and utility of mathematics. Problem solving is central to inquiry and application and should be interwoven throughout the mathematics curriculum to provide a context for learning and applying mathematical ideas. Middle-grades students whose curriculum is based on the Standards in this document will benefit from frequent opportunities for both independent and collaborative problem-solving experiences. They will engage profitably in complex

investigations, perhaps occasionally working for several days on a single problem and its extensions.

Problem solving in grades 6-8 should promote mathematical learning. Students can learn about, and deepen their understanding of, mathematical concepts by working through carefully selected problems that allow applications of mathematics to other contexts. Many interesting problems can be suggested by everyday experiences, such as reading literature or using cellular telephones, in-line skates, kites, and paper airplanes.

Instruction in grades 6-8 should take advantage of the expanding mathematical capabilities of students to include more-complex problems that integrate such topics as probability, statistics, geometry, and rational numbers. Situations and approaches should build on and extend the mathematical understanding, skills, and language that students have acquired.

Definition IX

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What Is and Is Not a Problem?

What is the definition of a problem? A problem is a situation in which a person is seeking some goal and for which a suitable course of action is not immediately apparent. In the context of the mathematics curriculum, a problem is one requiring that mathematical skills, concepts, or processes be used to arrive at the goal.

Whether a situation is a problem is an individual matter, depending on an individual's reaction or relationship to that situation. The individual must understand the situation and be interested in resolving it, yet there must be some block presenting immediate resolution. If these conditions do not exist, then a situation is not a problem for that individual.

What may be a problem for one student may not be a problem for another. Analyzing the definition produces four criteria that define mathematical problems for students:

CRITERIA FOR MATHEMATICAL PROBLEMS

1. There is a perplexing situation that the student understands.
2. The student is interested in finding a solution.
3. The student is unable to proceed directly toward a solution.
4. The solution requires use of mathematical ideas.

Definition X

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Most textbooks offer the teacher a very meager sense of what is involved in problem solving. Teachers' guides have a list of problem-solving strategies that coincide with those that appear about four times each in the students' texts. Some common strategies are draw a picture, make a list, look for a pattern, and so on. A set of problems is supposed to be worked on using a particular strategy. However, it is unclear how these strategies relate to one another or how they add up to what problem solving is. Merely giving students problems to work on is insufficient.

Students should experience problems that fit with the curriculum in order to initiate and deepen their understanding of key concepts. Simultaneously, students should be developing their repertoire of problem-solving strategies based on sound mathematical thinking. In order to establish a productive, dynamic relationship between students' schemata (their knowledge and how it is structured) and mathematical ideas, the teacher must carefully consider what kinds of problems make the best bridge. Some criteria of good problems are the following:

- There is no obvious way to work on the problem.
- The problem is set in a meaningful context for the students.
- The problem provokes students' interest in pursuing it.
- Working on the problem should use mathematical thinking and knowledge that is developmentally appropriate.
- Discussion of solutions should allow the teachers to build on the problem to explore mathematical ideas.

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For most students, the problems of their texts do not require much thought. They are "routine"—occasions merely to figure out what operation to perform. ... Teachers should step back from the textbook, workbook, and handouts and determine the key concepts of the curriculum at their grade level. Next they must similarly analyze the knowledge, abilities, motivations, interest, and schemata of the children they are teaching. Then, the vital step: inventing good problems or problem-solving activities that bring together the two entities: the domain of mathematics and the world of the students.