

Definition Set B

Definition I

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Problem Solving

Problem solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking.

By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. In everyday life and in the workplace, being a good problem solver can lead to great advantages.

Problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program. Problem solving in mathematics should involve all the five content areas described in these Standards. The contexts of the problems can vary from familiar experiences involving students' lives or the school day to applications involving the sciences or the world of work. Good problems will integrate multiple topics and will involve significant mathematics.

Definition II

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Problem Solving Standard for Grades 6 – 8

Through problem solving, students can experience the power and utility of mathematics. Problem solving is central to inquiry and application and should be interwoven throughout the mathematics curriculum to provide a context for learning and applying mathematical ideas. Middle-grades students whose curriculum is based on the Standards in this document will benefit from frequent opportunities for both independent and collaborative problem-solving experiences. They will engage profitably in complex

investigations, perhaps occasionally working for several days on a single problem and its extensions.

Problem solving in grades 6-8 should promote mathematical learning. Students can learn about, and deepen their understanding of, mathematical concepts by working through carefully selected problems that allow applications of mathematics to other contexts. Many interesting problems can be suggested by everyday experiences, such as reading literature or using cellular telephones, in-line skates, kites, and paper airplanes.

Instruction in grades 6-8 should take advantage of the expanding mathematical capabilities of students to include more-complex problems that integrate such topics as probability, statistics, geometry, and rational numbers. Situations and approaches should build on and extend the mathematical understanding, skills, and language that students have acquired.

Definition V

Burns, Marilyn. *About Teaching Mathematics*, Page 18. Copyright 2000 by Math Solutions Publications. Reprinted by permission. All rights reserved.

What Is and Is Not a Problem?

Do not confuse arithmetic exercises with problems. If a student meets a situation that calls for two-place multiplication and has not been taught the algorithm for this process, then this might be a problem for that student. Generally though, arithmetic exercises are used to provide students practice with using standard mathematical procedures. Although a child may have difficulty arriving at a correct arithmetic answer, deciding on the method for getting the answer is not usually part of the child's task. Executing specific procedures—the algorithms—is the child's task.

Textbooks often present word problems so that all those on the same page are solved with the same operation. The first one on the page may be a problem for a student. But if the student realizes that the same procedure can be used for all the others (and students are quick to notice this), the rest are not problems. They are merely exercises that do not require thinking or reasoning but rather are repetitive application of the same algorithms.

Problem-solving situations, on the other hand, each demand that the child develop a plan for solution as well as execute the plan. Problem-solving techniques do exist, but they are general approaches, not algorithms that can be routinely applied to specific problems. Although arithmetic is essential for solving many problems in life, it is not the only mathematical skill generally needed. Figuring how much floor covering is needed for a room requires applying both geometry and measurement skills. Deciding on the best place to put savings involves ideas in the areas of probability and statistics. Assembling a bicycle that arrives in a crate calls for the application of logical thinking skills.

Definition VI

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What Is Problem Solving?

It seems that everyone concerned with mathematics education today is talking about problem solving. Professional organizations are recommending that problem solving become the focus of school mathematics, curriculum guides are listing problems solving skills as key objectives at all levels, and it is difficult to find a meeting of educators that doesn't have at least one problem solving session on its agenda. However, we should be careful not to think of this interest in problem solving as just another "bandwagon." The ultimate goal of school mathematics at *all* times is to develop in our students the ability to solve problems.

Some teachers believe that the ability to solve problems develops automatically from mastery of computational skills. This is not necessarily true. Problem solving is itself a skill that needs to be taught, and mathematics teachers must make a special effort to do so.

Since we will be using the word "problem" repeatedly, let's begin by agreeing on its meaning. Any mathematical task can be classified as either an exercise or a problem. An *exercise* is a task for which a procedure for solving is already known; frequently an exercise can be solved by the direct application of one or more computational algorithms. A *problem* is more complex because the strategy for solving is not immediately apparent; solving a problem requires some degree of creativity or originality on the part of the problem solver.