

Definition Set A

Definition I

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Problem Solving

Problem solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking.

By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. In everyday life and in the workplace, being a good problem solver can lead to great advantages.

Problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program. Problem solving in mathematics should involve all the five content areas described in these Standards. The contexts of the problems can vary from familiar experiences involving students' lives or the school day to applications involving the sciences or the world of work. Good problems will integrate multiple topics and will involve significant mathematics.

Definition II

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Problem Solving Standard for Grades 6 – 8

Through problem solving, students can experience the power and utility of mathematics. Problem solving is central to inquiry and application and should be interwoven throughout the mathematics curriculum to provide a context for learning and applying mathematical ideas. Middle-grades students whose curriculum is based on the Standards in this document will benefit from frequent opportunities for both independent and collaborative problem-solving experiences. They will engage profitably in complex

investigations, perhaps occasionally working for several days on a single problem and its extensions.

Problem solving in grades 6-8 should promote mathematical learning. Students can learn about, and deepen their understanding of, mathematical concepts by working through carefully selected problems that allow applications of mathematics to other contexts. Many interesting problems can be suggested by everyday experiences, such as reading literature or using cellular telephones, in-line skates, kites, and paper airplanes.

Instruction in grades 6-8 should take advantage of the expanding mathematical capabilities of students to include more-complex problems that integrate such topics as probability, statistics, geometry, and rational numbers. Situations and approaches should build on and extend the mathematical understanding, skills, and language that students have acquired.

Definition III

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There is wide consensus that schools should shift their focus from memorizing basic skills to promoting understanding and problem solving (NCTM, 1989). ...

To develop mathematical thinking and the autonomy to solve challenging mathematical problems, children need to *do* mathematics (National Research Council, 1989, 1990). Doing mathematics here does not mean doing rows and rows of computational “problems.” It entails:

- Solving challenging problems;
- Exploring patterns;
- Formulating educated guesses (conjectures) and checking them out;
- Drawing conclusions (reasoning); and
- Communicating ideas, patterns, conjectures, conclusions, and reasons.

...In brief, whether a task constitutes a genuine problem depends on (a) a child’s readiness and (b) the attractiveness of the task to the child (Charles & Lester, 1982).

To ensure that tasks are genuine problems—challenging but not overwhelming—teachers must constantly evaluate their students’ thinking and interests.

...Word problems that require a thoughtful analysis to devise a *new* strategy or to use existing strategies in *new* ways would constitute a genuine problem. Those that require a thoughtful analysis to decide among existing strategies ... might also be considered genuine problems in the sense that they lack an obvious solution procedure. Word

problems in which a child can readily identify and apply a solution strategy are more like routine practice exercises. Unfortunately, this is how word problems have historically been used in instruction.

Definition IV

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We are not merely suggesting that problem solving should be the culmination of learning a mathematical idea, the application of a known concept. Problem solving should not be a set of problems at the end of a chapter. Because problem solving is a way of thinking and doing mathematics, it can be a major vehicle for helping students truly understand and use mathematics. Problem-solving activities can introduce students to new mathematical ideas, provide exciting experiences that develop deeper understanding, and also help students apply what they understand to their lives.

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Is it really possible to go through the entire curriculum creating problem-solving activities for every concept? Yes, and it is not as mammoth a task as one might imagine. First, the number of fundamental concepts to be taught in a year is large but not enormous. Second, some concepts require less experiential problem-solving activities than others, for two reasons: (1) students may already have good conceptions . . . to build upon, and (2) many concepts can be related to one another.

We want students to develop substantial mathematical knowledge and make sense of the world around them. We want students to be engaged and motivated, to become self-reliant, to feel competent and capable of generating and investigating questions, collecting and analyzing information, and solving problems in school and in life. When properly balanced, teaching problem solving can enable us to accomplish all of these mutually compatible goals.