In the game of Chuck-a-Luck three fair dice are rolled. You as the “bettor” are allowed to bet $1 on the occurrence of one of the integers 1, 2, 3, 4, 5, or 6. Suppose you bet on the occurrence of a “5”. Then if any 5’s occur on the 3 dice, you get your $1 bet back. In addition, if one 5 occurs (on the 3 dice) you win $1, if two 5’s occur you win $2, and if three 5’s occur you win $3. If no 5’s occur you lose your dollar.

1. In the long run, what amount of money would you expect to be paid (or lose) for one bet in the Chuck-a-Luck Game?

2. If you played 100 games, how much money would you expect to win (or lose)? Why?

3. Is the Chuck-a-Luck Game fair?
Tree Diagram of Chuck-a-Luck Game

\[
P(X=3) = \frac{1}{6}^3 \left(\frac{5}{6}\right)^0 \\
P(X=2) = \frac{1}{6}^2 \left(\frac{5}{6}\right)^1 \\
P(X=2) = \frac{1}{6}^2 \left(\frac{5}{6}\right)^1 \\
P(X=1) = \frac{1}{6}^1 \left(\frac{5}{6}\right)^2 \\
P(X=2) = \frac{1}{6}^2 \left(\frac{5}{6}\right)^1 \\
P(X=1) = \frac{1}{6}^1 \left(\frac{5}{6}\right)^2 \\
P(X=0) = \frac{1}{6}^0 \left(\frac{5}{6}\right)^3 \\
\]

\[
P(X=3) = \frac{1}{216} \quad P(X=2) = \frac{15}{216} \quad P(X=1) = \frac{75}{216} \quad P(X=0) = \frac{125}{216}
\]

\[
E(X) = -$1 \left(\frac{125}{216}\right) + $1 \left(\frac{75}{216}\right) + $2 \left(\frac{15}{216}\right) + $3 \left(\frac{1}{126}\right) = -$\left(\frac{17}{216}\right) \approx -$0.08
\]

On average, you should expect to lose approximately $0.08 each time you play a round of the game. In approximately 100 games, one should expect to lose $8.

Although the first statement does not make much sense, it does provide us with information that allows us to make predictions about what will happen in the long run.