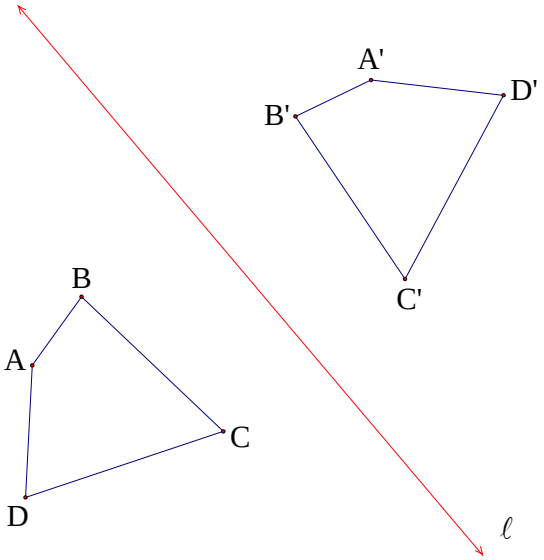
Solutions to Exercises

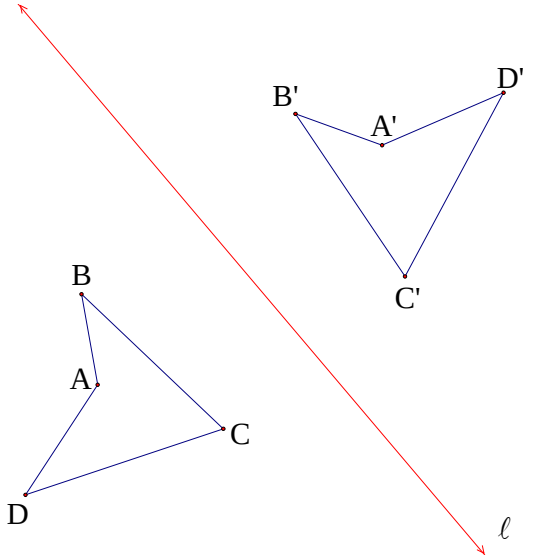
Module 4: Geometry and Spatial Reasoning
Section: Transformations

Solutions to Reflections Exercises: (Non-Coordinate Plane)

1.



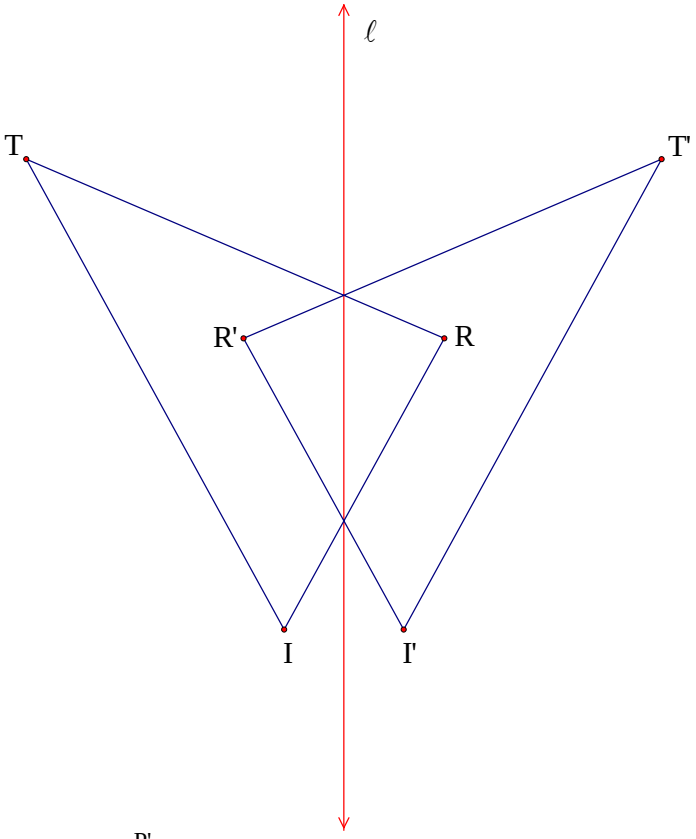
2.



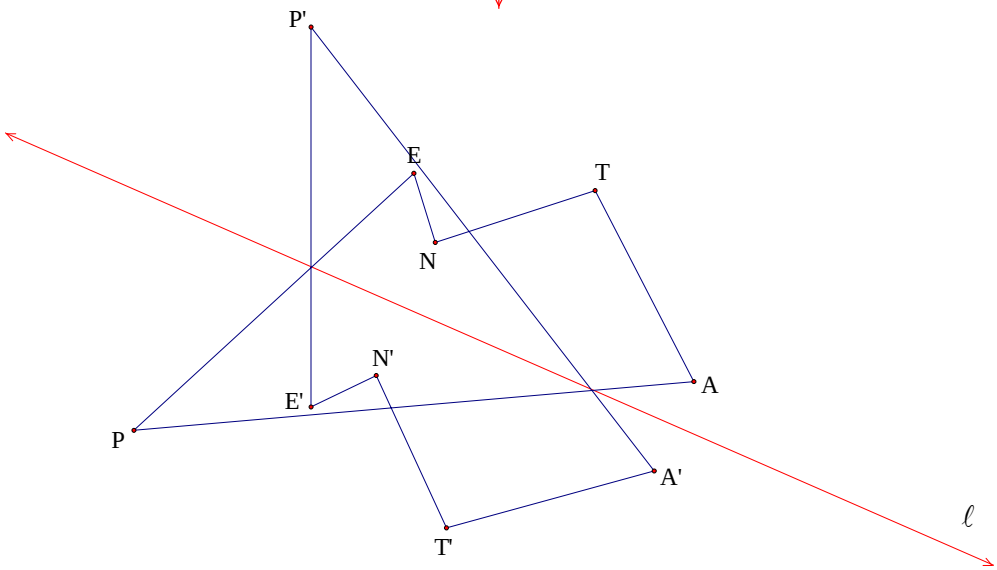
Solutions to Reflections Exercises: (Non-Coordinate Plane)

3. Individual answers vary.

4.



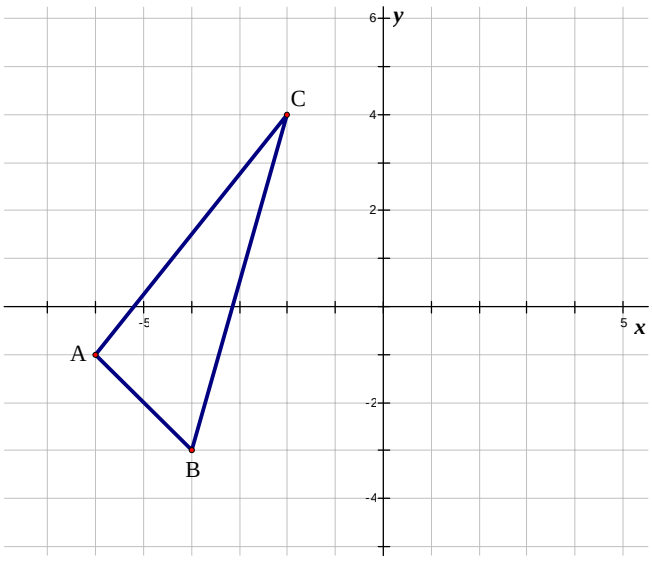
5.



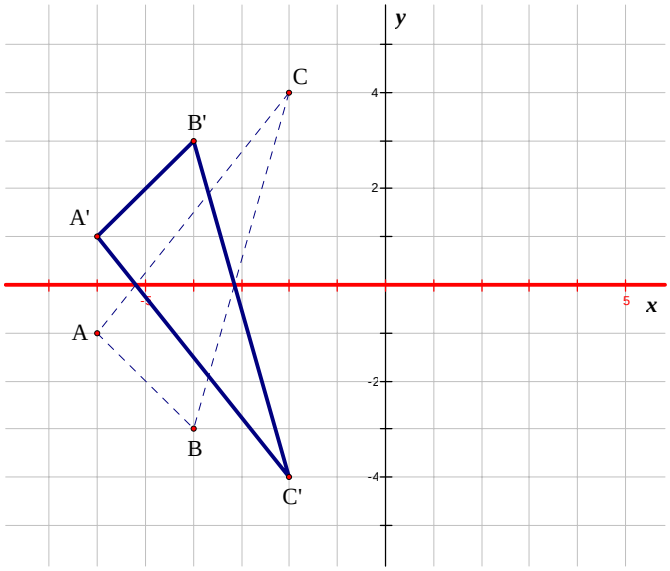
6. Individual answers vary.

Solutions to Reflections Exercises: (Coordinate Plane)

1.



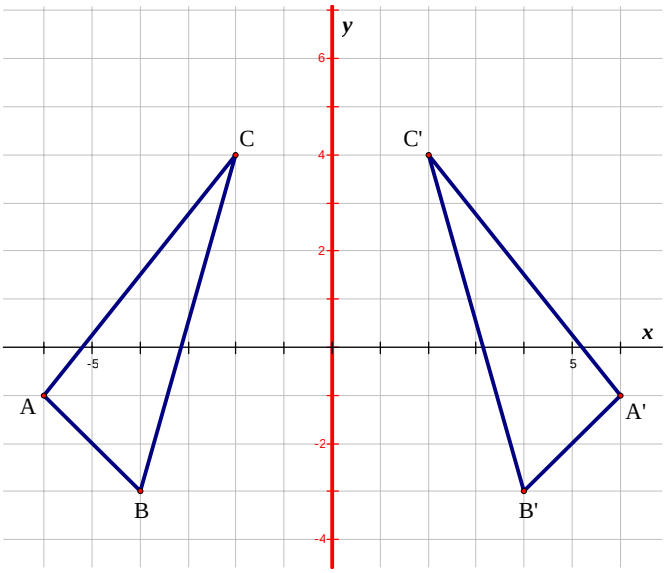
2. a)



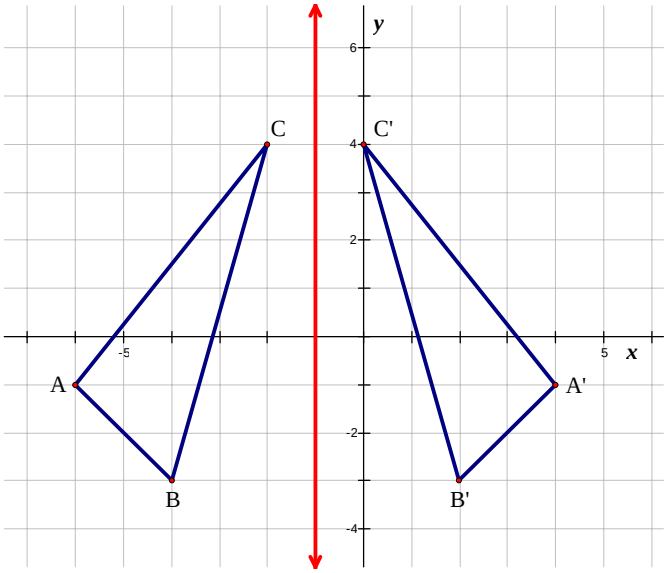
The segments of the pre-image are dashed to avoid confusion between the pre-image and the image.

Solutions to Reflections Exercises: (Coordinate Plane)

2. b)

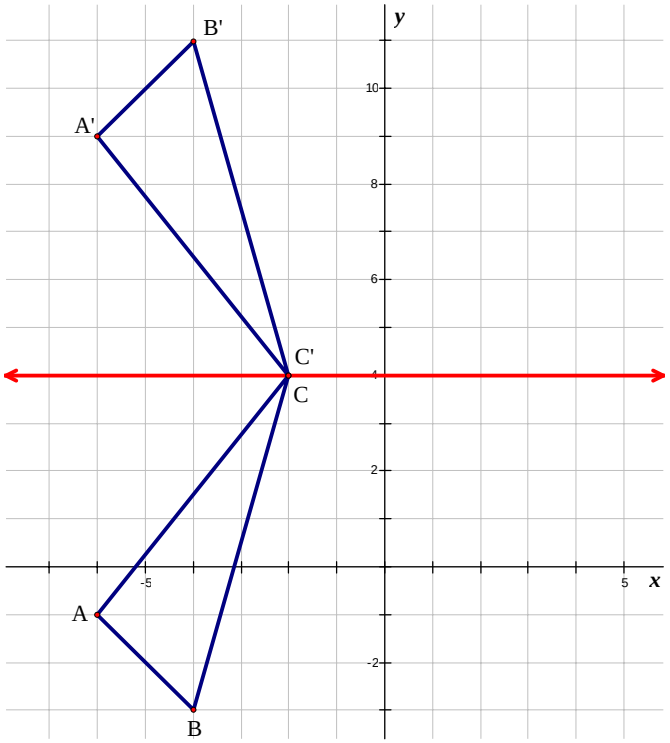


c)

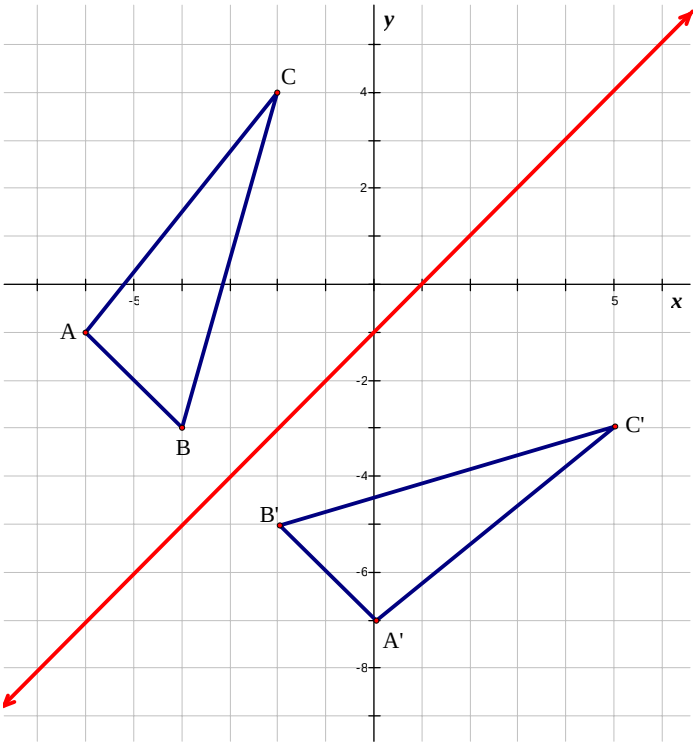


Solutions to Reflections Exercises : (Coordinate Plane)

2. d)

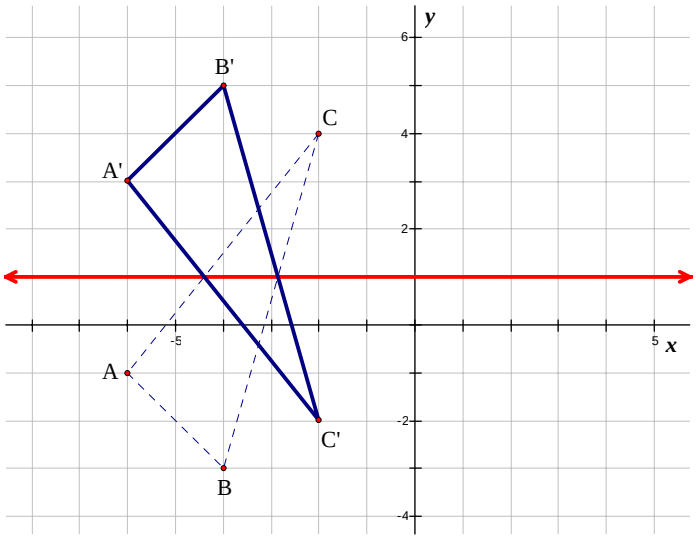


e)



Solutions to Reflections Exercises : (Coordinate Plane)

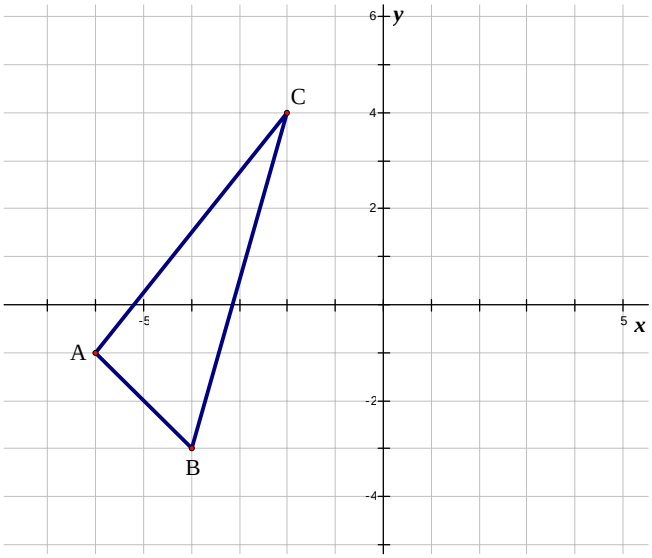
2. f)



The segments of the pre-image are dashed to avoid confusion between the pre-image and the image.

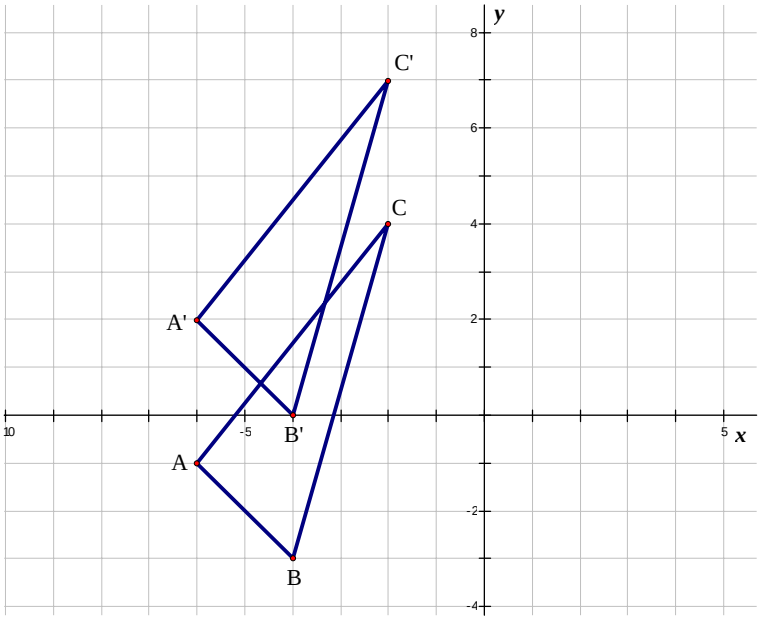
Solutions to Translations Exercises: (Coordinate Plane)

1.

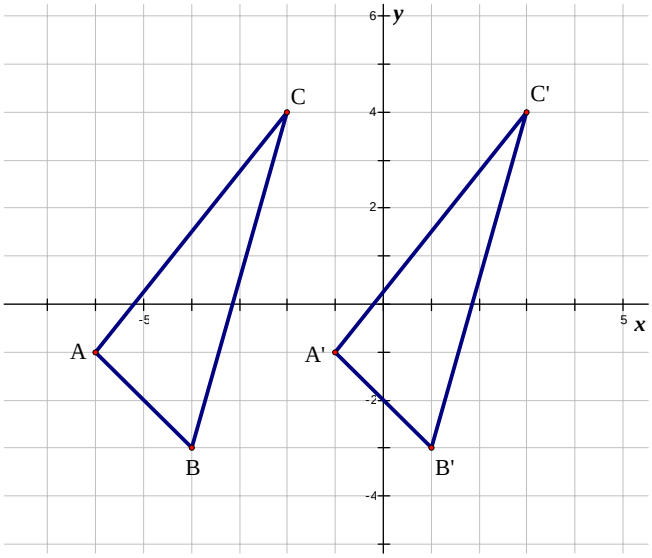


Solutions to Translations Exercises: (Coordinate Plane)

2. a)

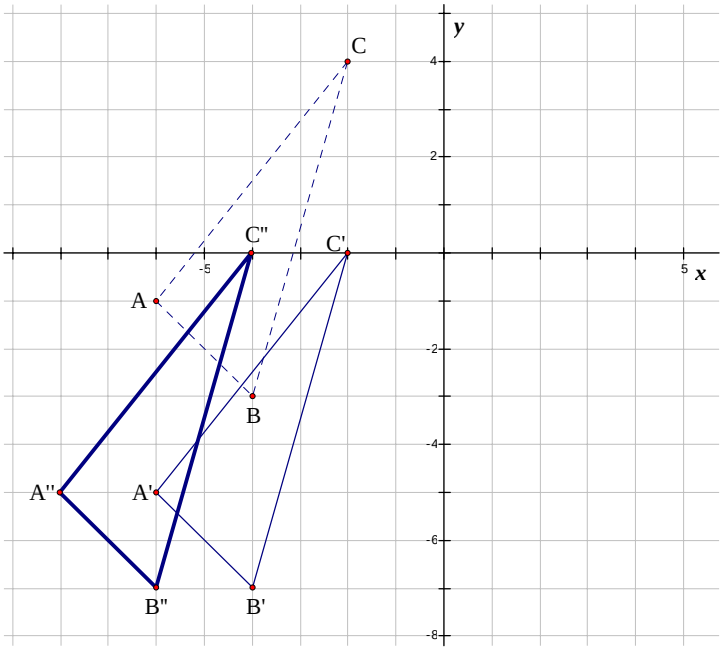


b)



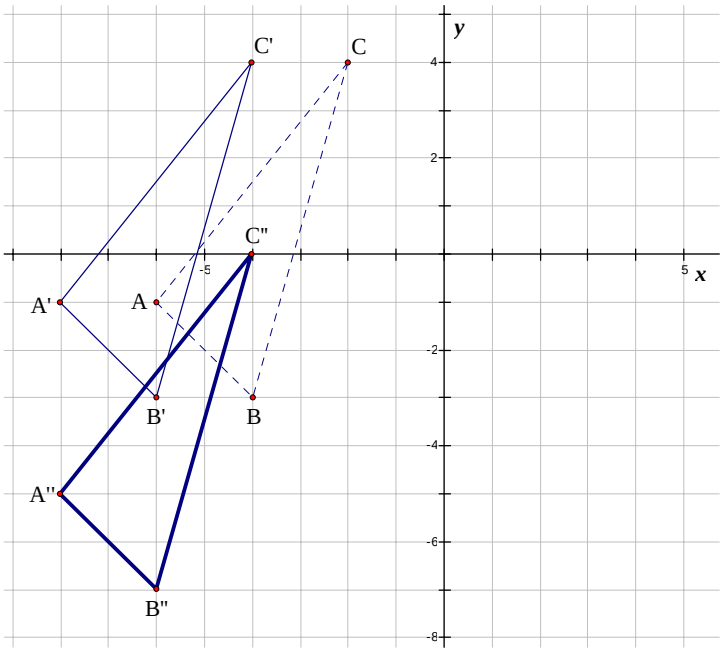
Solutions to Translations Exercises: (Coordinate Plane)

3. a)



The segments of the pre-image (ΔABC) are dashed, the segments of the intermediate image (ΔA'B'C') are thin, and the segments of the final image (ΔA"B"C") are thick.

b)

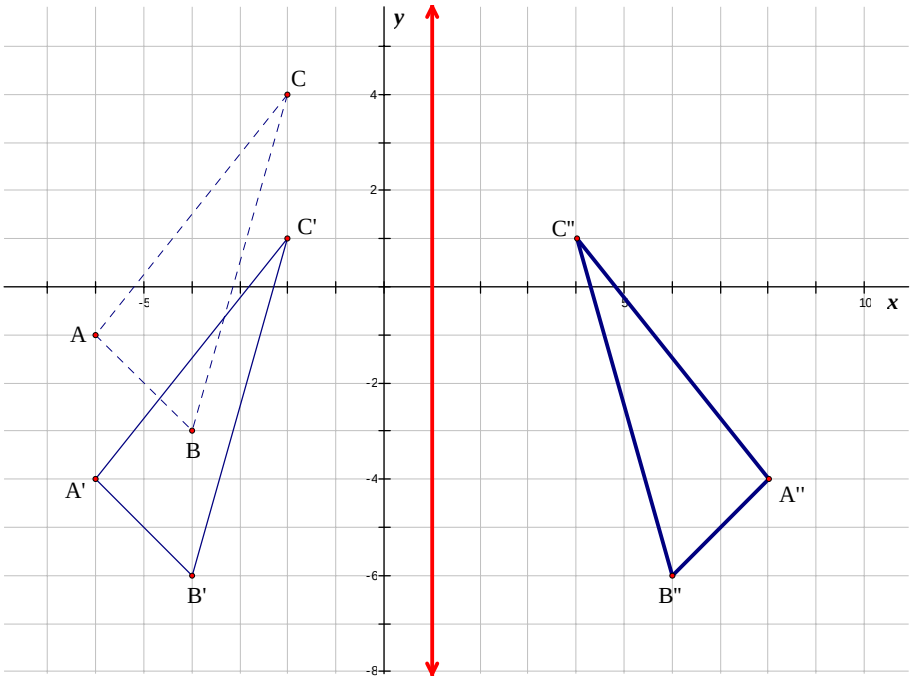


The segments of the pre-image (ΔABC) are dashed, the segments of the intermediate image (ΔA'B'C') are thin, and the segments of the final image (ΔA"B"C") are thick.

c) Yes, the final images in parts (a) and (b) are identical. Translating four units down and two units to the left yields the same result as translating two units to the left and four units down.

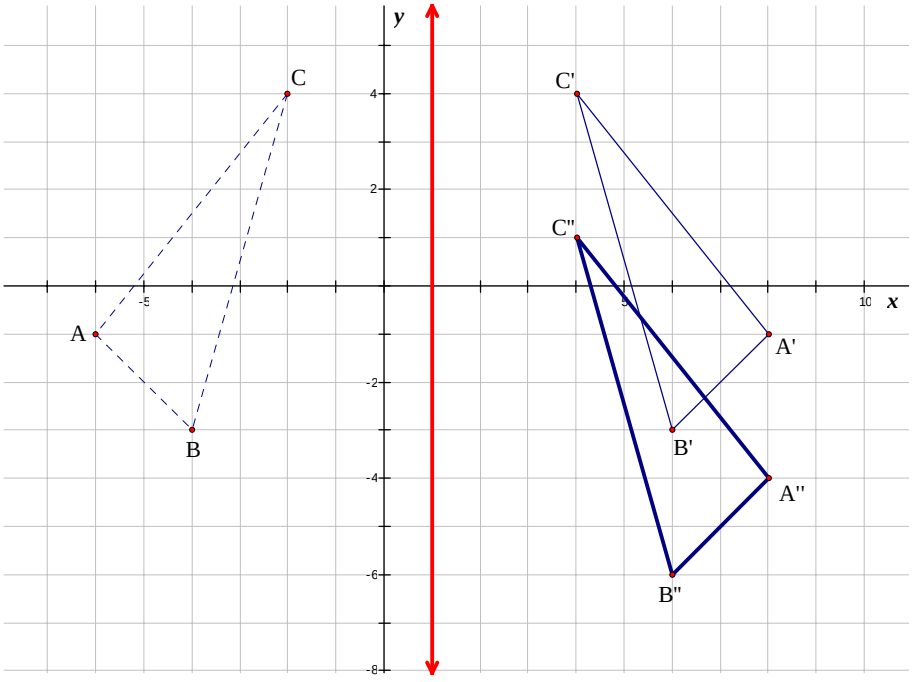
Solutions to Translations Exercises: (Coordinate Plane)

4. a)



The segments of the pre-image (ΔABC) are dashed, the segments of the intermediate image (ΔA'B'C') are thin, and the segments of the final image (ΔA"B"C") are thick.

b)



The segments of the pre-image (ΔABC) are dashed, the segments of the intermediate image (ΔA'B'C') are thin, and the segments of the final image (ΔA"B"C") are thick.

Solutions to Translations Exercises: (Coordinate Plane)

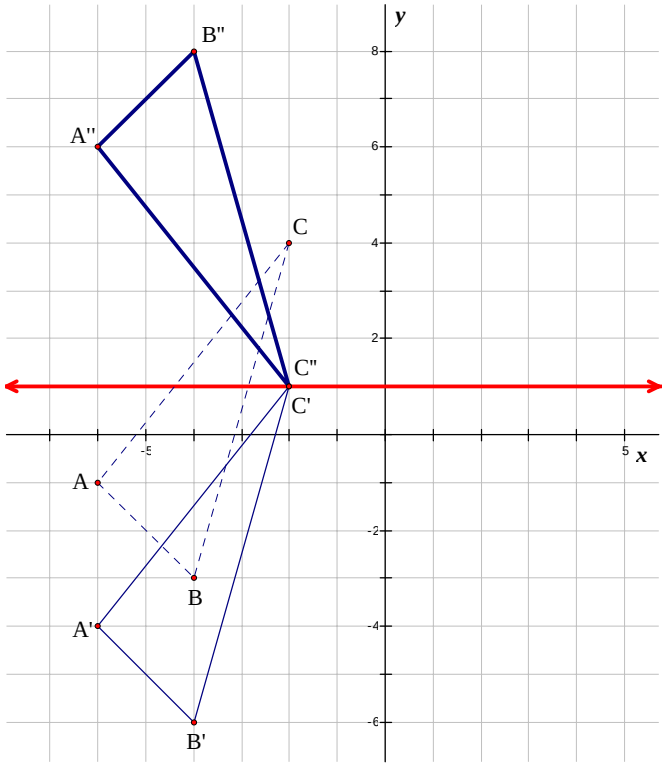
4. c) Yes, the final images in parts (a) and (b) are identical. Translating three units down and then reflecting across the line $x = 1$ yields the same result as reflecting across the line $x = 1$ and then translating three units down.

   d) Translating any object down three units results in a change in its vertical position.

   e) Reflecting any object across the line $x = 1$ results in a change in its horizontal position.

   f) When a vertical translation (translating up or down) is combined with a horizontal reflection (reflecting across any vertical line), changing the order of the two transformations yields the same result. An extension to this statement (which was not illustrated in the diagrams above) is that when a horizontal translation (translating left or right) is combined with a vertical reflection (reflecting across any horizontal line), changing the order of the two transformations yields the same result.

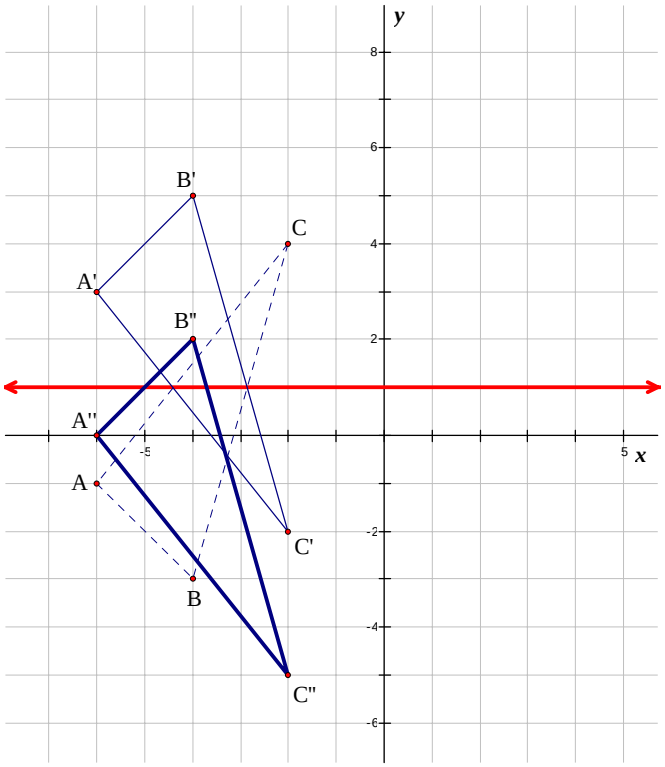
5. a)



The segments of the pre-image (ΔABC) are dashed, the segments of the intermediate image ($\Delta A'B'C'$) are thin, and the segments of the final image ($\Delta A''B''C''$) are thick.

Solutions to Translations Exercises: (Coordinate Plane)

5. b)



The segments of the pre-image (ΔABC) are dashed, the segments of the intermediate image ($\Delta A'B'C'$) are thin, and the segments of the final image ($\Delta A"B"C"$) are thick.

c) No, the final images in parts (a) and (b) are in different locations (although they are congruent). Translating three units down and then reflecting across the line $y = 1$ yields a different result than reflecting across the line $y = 1$ and then translating three units down.

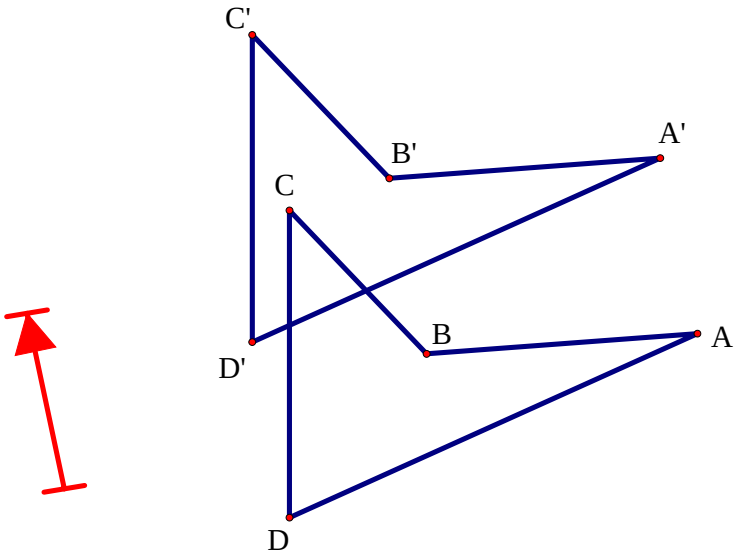
d) Translating any object down three units results in a change in its vertical position.

e) Reflecting any object across the line $y = 1$ results in a change in its vertical position.

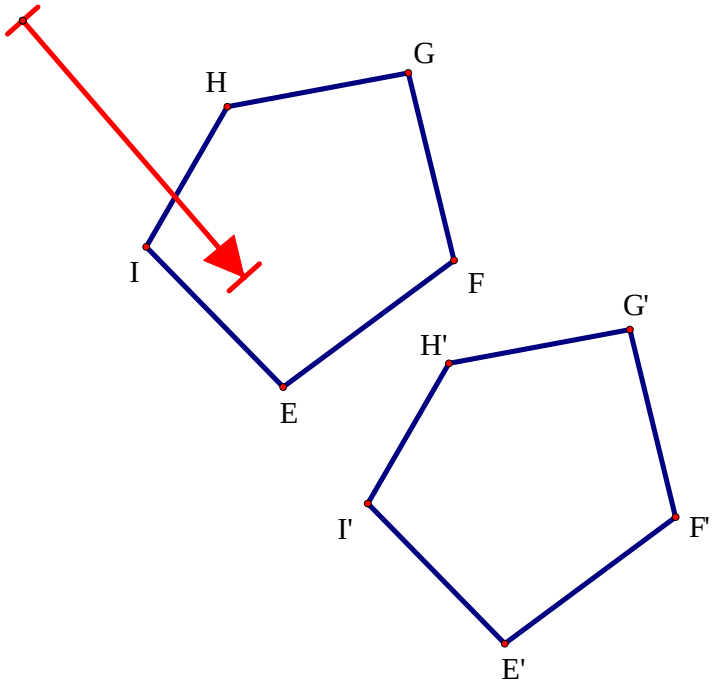
f) When a vertical translation (translating up or down) is combined with a vertical reflection (reflecting across any horizontal line), changing the order of the two transformations yields two different results. An extension to this statement (which was not illustrated in the diagrams above) is that when a horizontal translation (translating left or right) is combined with a horizontal reflection (reflecting across any vertical line), changing the order of the two transformations yields two different results.

Solutions to Translations Exercises: (Non-Coordinate Plane)

1. a)

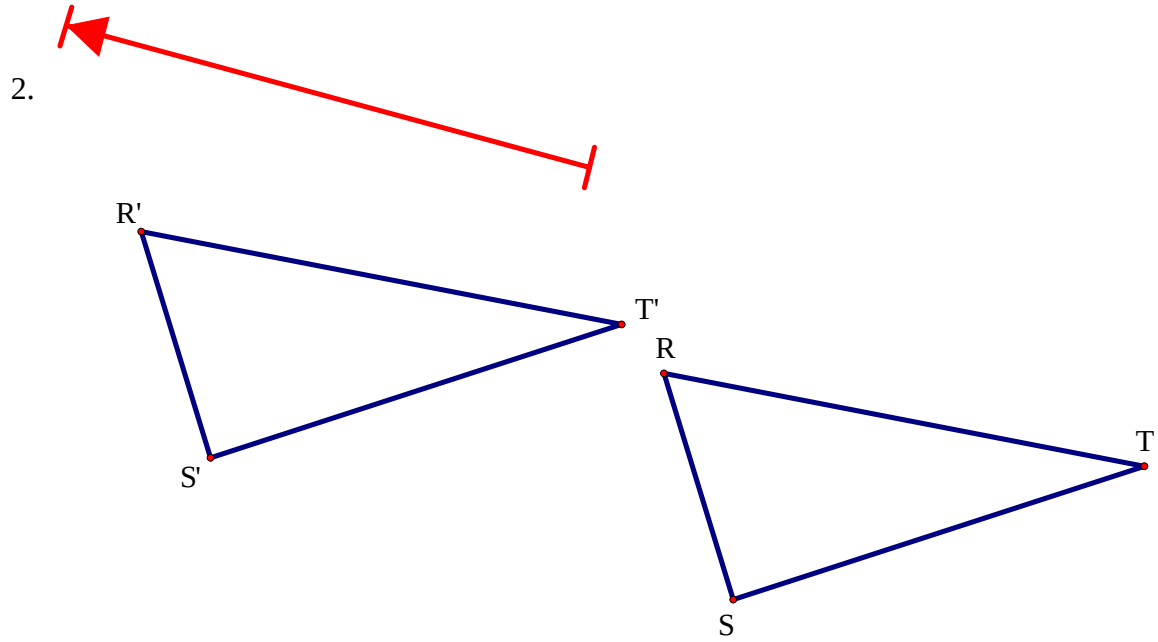


b)

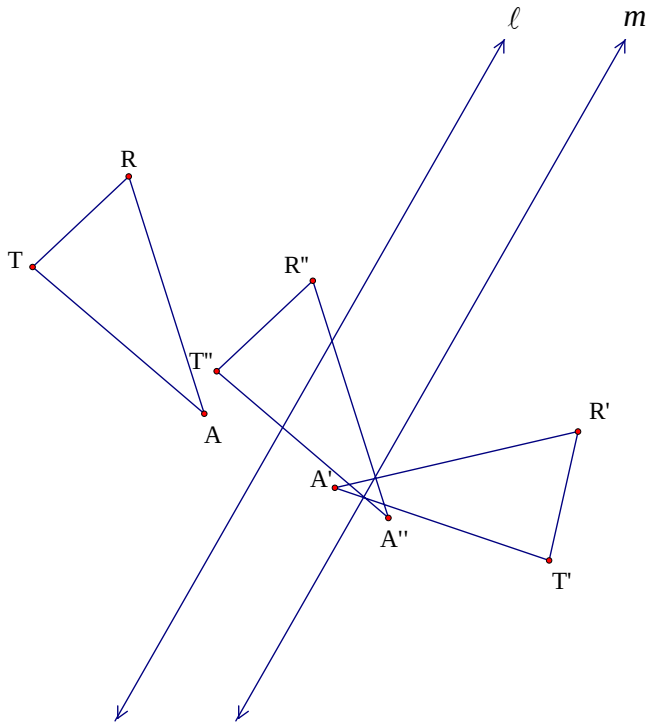


Solutions to Translations Exercises: (Non-Coordinate Plane)

2.

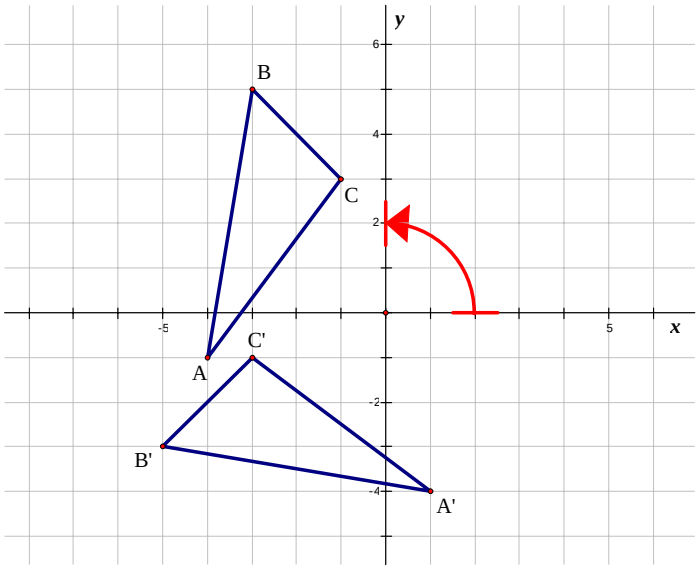


3. Answers vary. One possible solution is shown below. We notice that $\Delta T''R''A''$ is a translation of ΔTRA. We can be more specific and describe the translation vector (not shown in the diagram below): The direction of the translation vector is perpendicular to lines ℓ and m, and the length of the translation vector is twice that of the distance between lines ℓ and m.

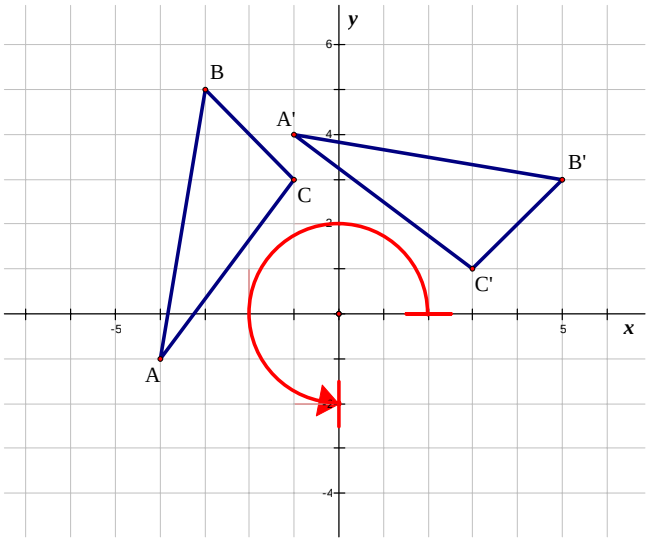


Solutions to Rotations Exercises:

1. a)

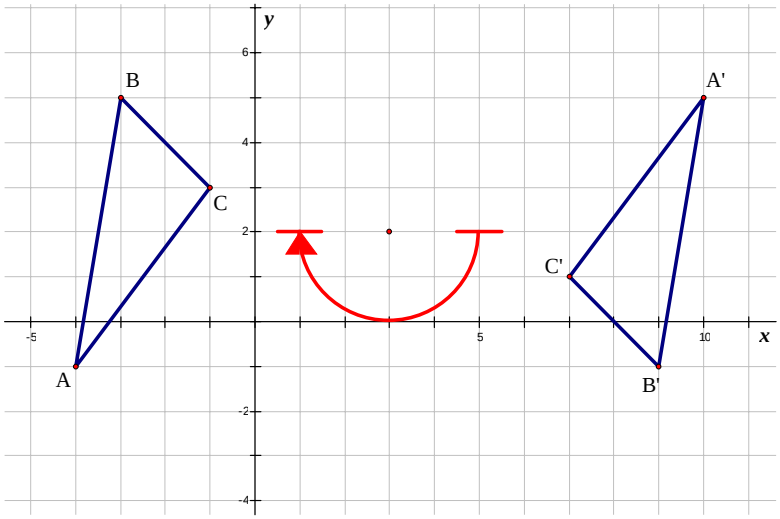


b)

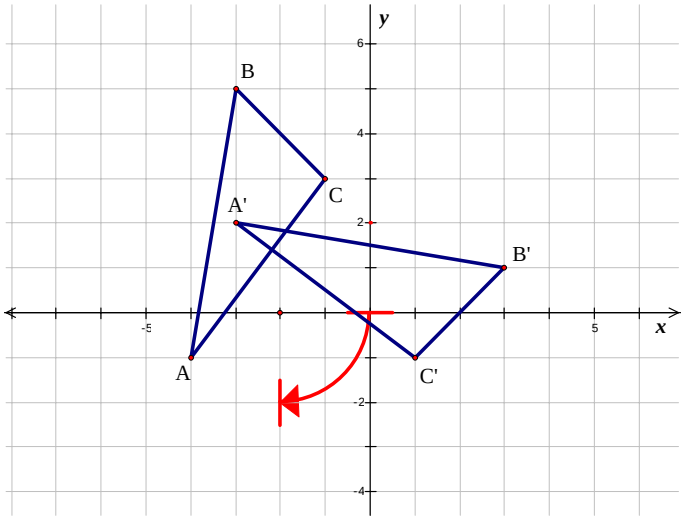


Solutions to Rotations Exercises:

1. c)

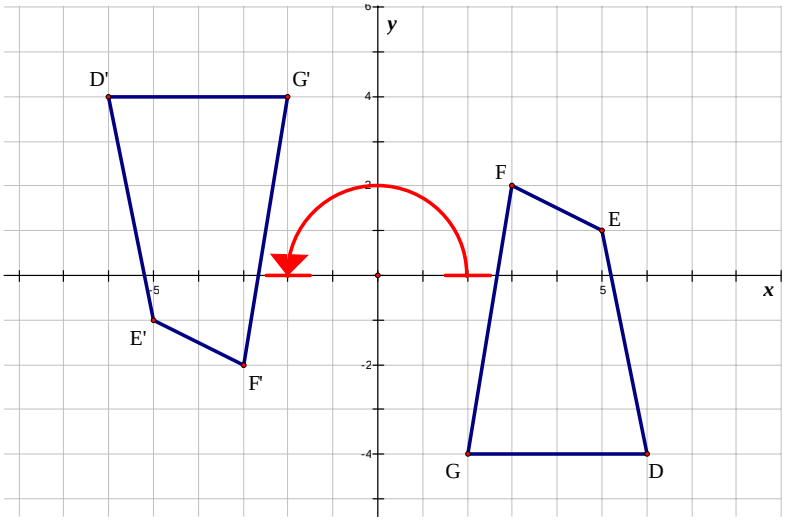


d)

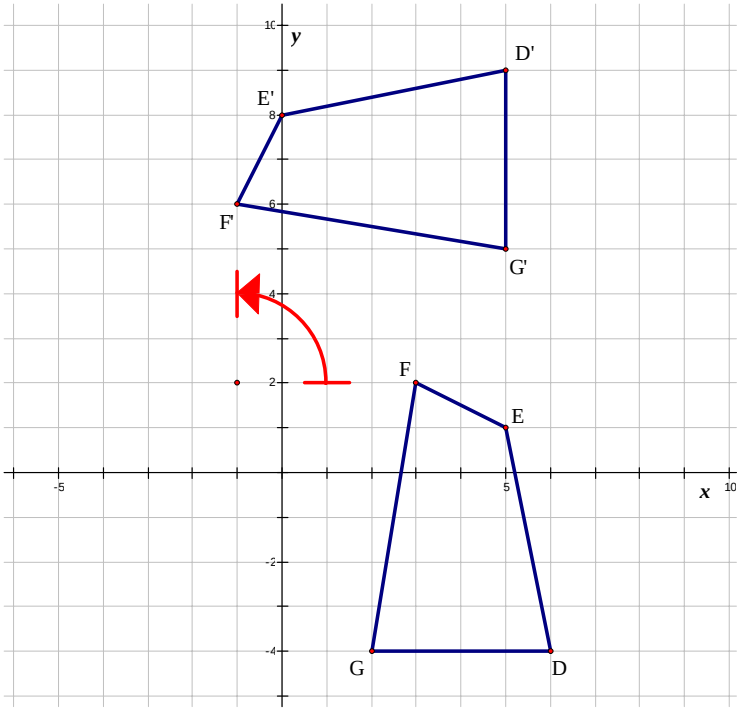


Solutions to Rotations Exercises:

2. a)

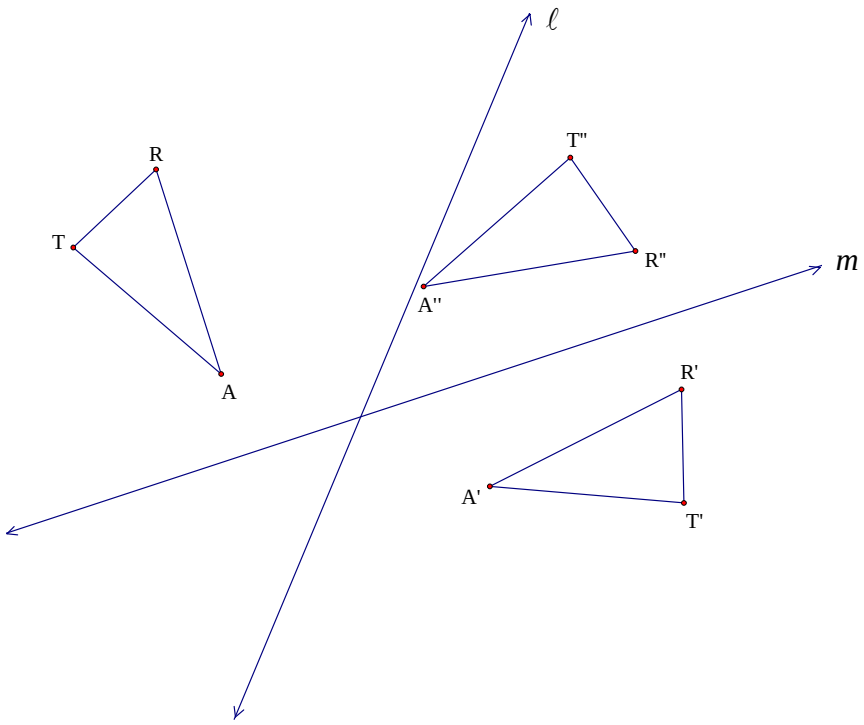


b)



Solutions to Rotations Exercises:

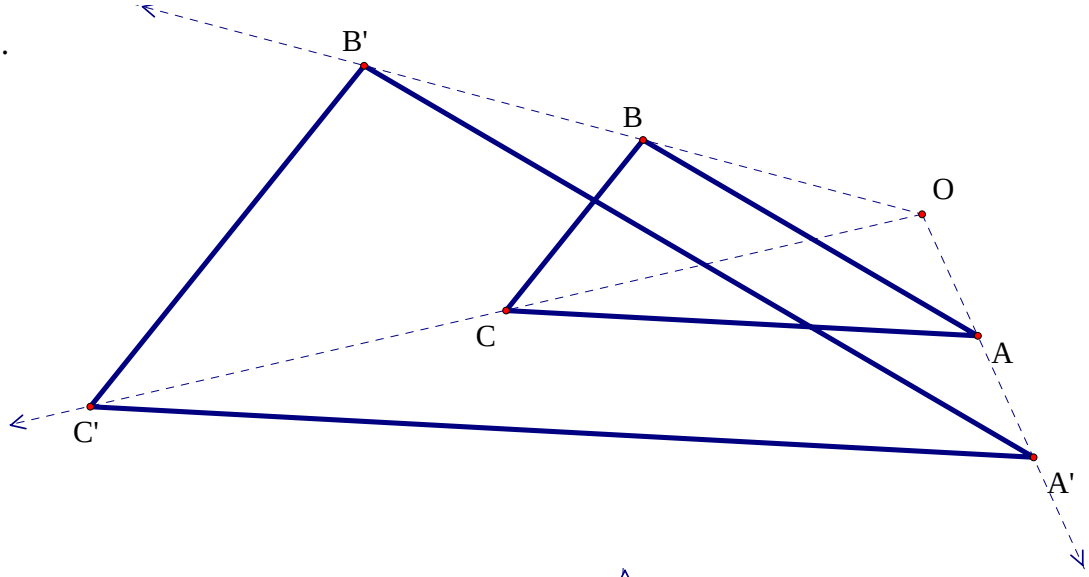
3. Answers vary. One possible solution is shown below. We notice that $\Delta T"R"A"$ is a rotation of ΔTRA. We can be more specific and describe the rotation arrow (not shown in the diagram below): The center of the rotation arrow is the intersection of lines ℓ and m, and the amount of rotation is twice the angle formed by the two intersecting lines. In this particular diagram, the acute angle formed by lines ℓ and m in the figure below measures 49.1° (clockwise if measuring from ℓ to m), and the amount of rotation is $2(49.1^\circ) = 98.2^\circ$ clockwise. If the obtuse angle formed by lines ℓ and m is instead measured, the obtuse angle measures 130.9° (counterclockwise from ℓ to m), resulting in a counterclockwise rotation of $2(130.9^\circ) = 261.8^\circ$. (Note that a clockwise rotation of 98.2° is equivalent to a counterclockwise rotation of 261.8°, since $360^\circ - 98.2^\circ = 261.8^\circ$.)



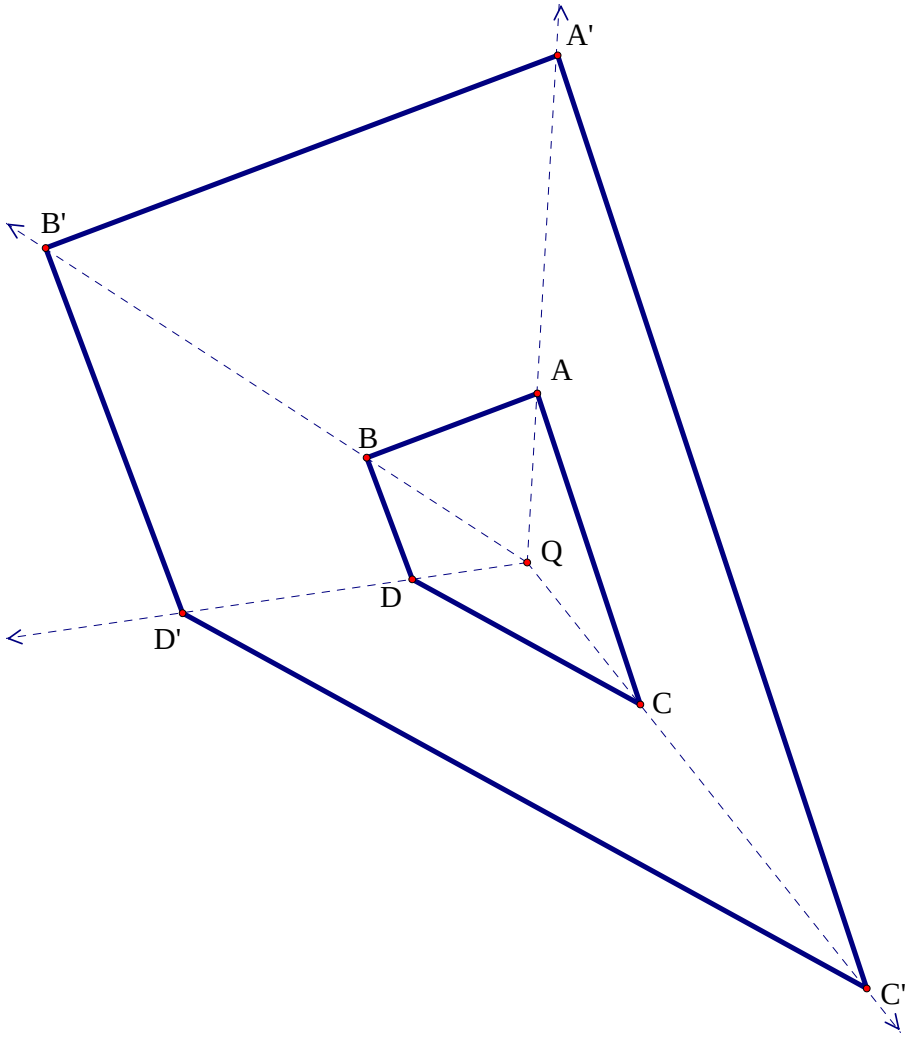
Solutions to Dilations Exercises: (Non-Coordinate Plane)

Note: The dashed dilation rays are not necessary for the final solution, but are drawn to help the reader to understand how the dilation was constructed.

1.

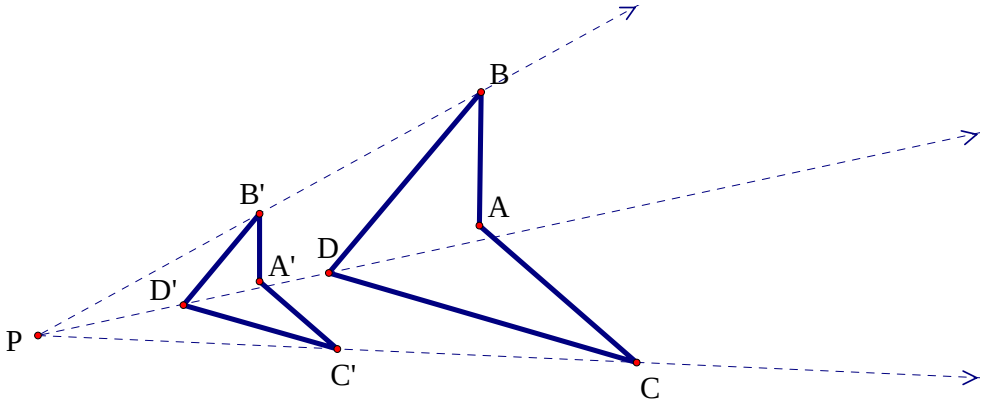


2.

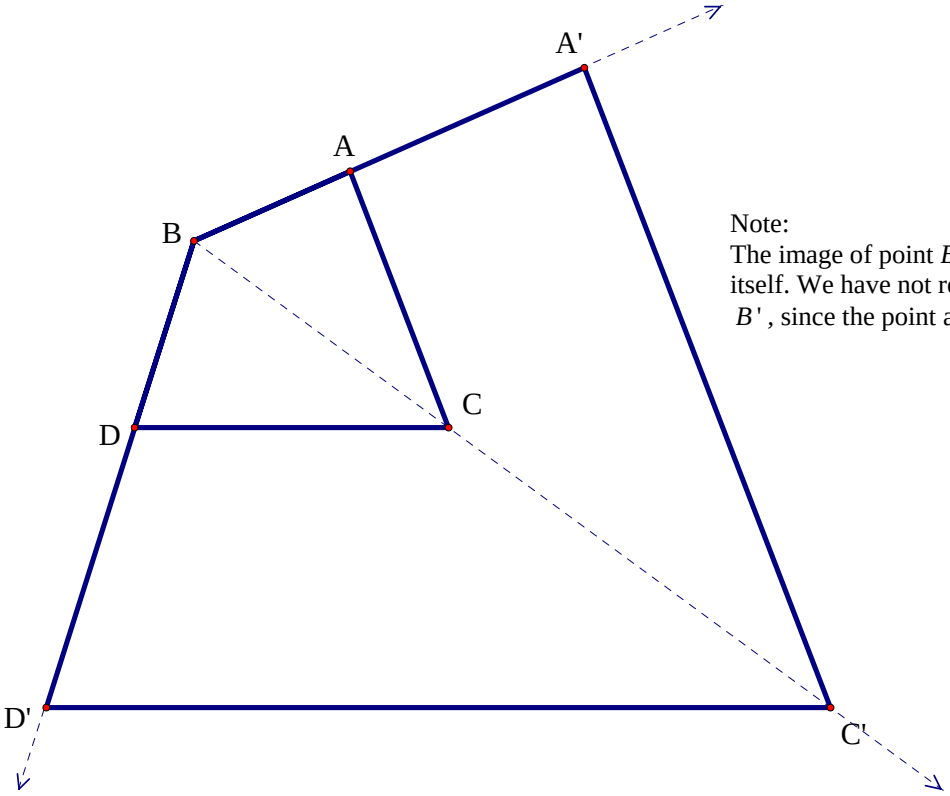


Solutions to Dilations Exercises: (Non-Coordinate Plane)

3.



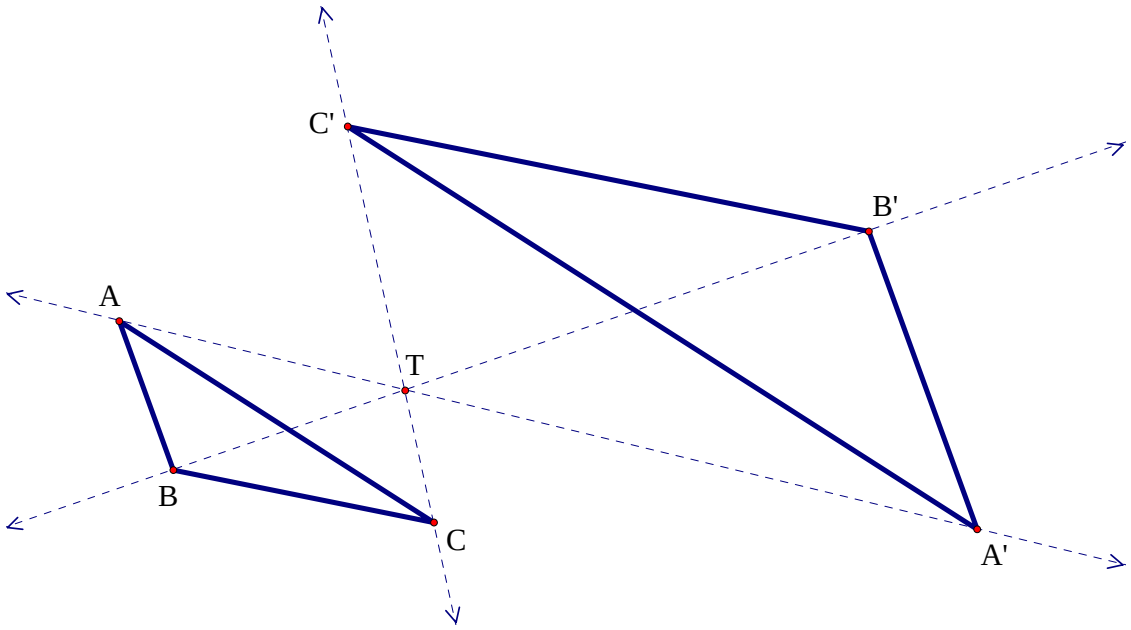
4.



Note:
The image of point *B* falls on top of itself. We have not re-named this point as *B'*, since the point already has a name.

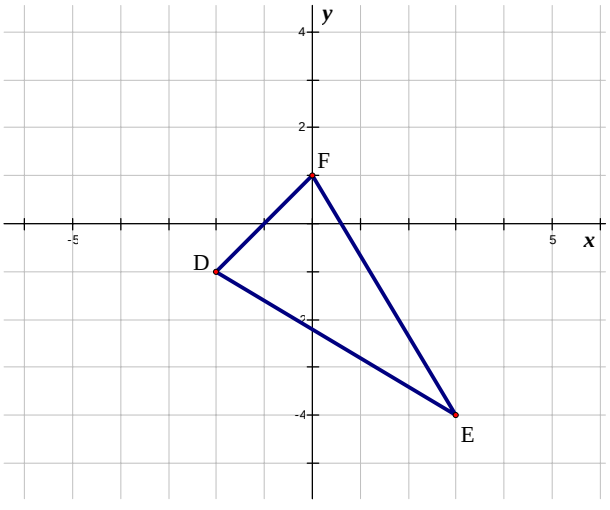
Solutions to Dilations Exercises: (Non-Coordinate Plane)

5



Solutions to Dilations Exercises: (Coordinate Plane)

1.



Solutions to Dilations Exercises: (Coordinate Plane)

2. $D'(-6,-3)$, $E'(9,-12)$, and $F'(0,3)$

3.