CONSTRUCTIONS

The following constructions are included in this unit:

Construction #1: Congruent segments
Construction #2: Congruent angles
Construction #3: Parallel lines

Many constructions can be done in a variety of ways, so these constructions are not necessarily unique.

Each construction will provide the following information:

A. Constructing the figure using a straightedge and compass

   1. Detailed instructions on how to construct the figure.

   2. A diagram of the construction.

   3. A justification of why the construction works.

B. Constructing the figure using *The Geometer's Sketchpad* software

   1. Detailed instructions on how to construct the figure. (It is assumed that the reader has a basic knowledge of *The Geometer's Sketchpad*.)

   2. A diagram of the construction.

   3. A justification of why the construction works.

Although each construction is shown in two ways (straightedge and compass versus *The Geometer's Sketchpad*), the steps and justifications of the constructions are equivalent to each other. The only difference in the final diagrams is that entire circles are drawn in *Sketchpad,* but only arcs of circles are drawn with the straightedge and compass constructions.

To save time within the instructions themselves, a few comments are listed below in regard to the labeling, naming, re-naming and hiding of objects. At the end of this discussion, we also mention how to select multiple objects within *Sketchpad*.

When you draw an object within *The Geometer's Sketchpad*, the object itself is drawn but is not labeled. We may give instructions within a construction that say, for example, "Draw segment AB." It is then implied that you will first draw a segment, and then take additional steps to make sure that each of the points A and B are labeled correctly. Instructions on labeling objects (and changing the labels of objects) are listed below.

Note: It is possible to change the display preferences of *Sketchpad* to label points automatically as you proceed through a construction. We will mention below how to do this, but realize that the program may then display some labels that you do not wish to see, and you may need to hide them.

To display the label of an object within *Sketchpad* (points, lines, circles, etc.):
To display the names of objects within *Sketchpad* (such as point A, line l, circle C_1, etc.), click on the text tool (the "*A*" on the toolbar on the left) and then click on the object for which you want to see a label. Its label will then be displayed (and alternately hidden/displayed as you continue to click on that object). *Sketchpad* may give a name which you want to change (for example, a point may be labeled as "D" when you want to call it "B"). Instructions on changing labels of objects can be found below.

To change the label of an object within *Sketchpad*:
Once a label is displayed, you can change the label easily if you want the object to have another name. First, make sure that you are using either the selection tool (the arrow) or the text tool (the "*A*") from the toolbar on the left side of the screen. Then double click on the label that you wish to change. A dialog box should appear on the screen which shows the current label, and you can re-type a new name for the label.

To hide the label of an object within *Sketchpad*:
To hide the label of an object, first click on the text tool (the "*A*" on the toolbar on the left) and then click on the object whose label you wish to hide. (To hide point "B," for example, click on the point itself – not the label "B.") The label will be alternately hidden/displayed as you continue to click on that object.

To automatically display the labels of points within *Sketchpad*:
You can set up *The Geometer's Sketchpad* to automatically assign a label to each point as soon as it is created. This works for points only (such as A, B, C…); it does not automatically name lines (such as l), circles (such as C_1), etc. To automatically display the label of all points, do the following:

Click on the "Edit" menu,
then click on "Preferences"
then click on "Text"
then check the box "For All New Points."

Notice that a box is already checked on this screen that applies this rule to "This Sketch." If you would like for *Sketchpad* to label points in this way every time you use the program, then you should also check the box entitled "New Sketches."

Since this second option displays ALL labels for points, there may be some labels which you do not wish to see, in which case you can hide them.

To hide an object within *Sketchpad* (This is to hide the entire object – not just its label):
There are times within constructions when we will ask you to hide an object. For example, you may be asked to draw a line. When *Sketchpad* draws a line, it automatically displays a line and two points (since two points determine a line). We may ask you to hide one or both of the points if they are distracting and not needed for the rest of the construction. To hide an entire object (not just its label), do the following:

Using the selection tool (the arrow), select the object that you want to hide. Then go to the "Display" menu and click on "Hide Point," "Hide Line," etc. (Only one choice will be displayed as to what object you can hide; it corresponds to the type of figure that you have selected.) This will hide the object along with its label. Remember that this is not a delete function; the object is still there – but is hidden. (To unhide objects, click on "Display," then on "Show All Hidden.")

To select multiple objects:
There are times within *Sketchpad* when we will want to select more than one object. (For example, if you want to construct a circle by its center and radius, you need to select both the center and the radius.) to select multiple objects, click on the selection (arrow) tool, click on a 'blank' part of the screen (to deselect anything that is already selected) and then HOLD DOWN THE SHIFT KEY as you select your objects one by one.

Note about the naming of circles:
Since the convention in Geometry is to name a circle by its center, we will name circles by their center in the instructions for each construction (rather than clicking on the circle itself with the text tool to give the circle a name such as C_1, C_2, etc.). The only exception to this naming system will be if two circles have the same center, in which case we need to distinguish them from each other and will take the time to label them separately.
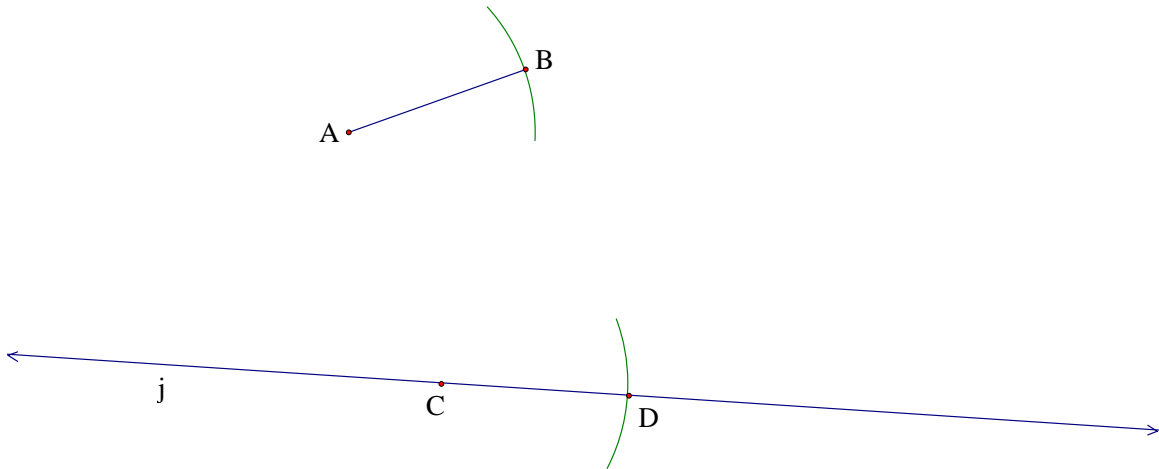
We now proceed to the constructions…

Construction #1A: Congruent Segments – Using a Straightedge and Compass

<u>Instructions:</u>

1. Use your straightedge to draw any segment $\overline{AB}$. (The purpose of the construction is now to construct a segment congruent to $\overline{AB}$.)

2. Use your straightedge to draw a line. Label it *j*.

3. Place a point *C* on line *j*.

4. Place the center of your compass on point *A* and draw an arc through point *B*.

5. Keeping the compass open to the same radius, place the center of your compass on point *C* and draw an arc which intersects line *j*. Label this point of intersection *D*.

We have now completed our construction; $\overline{AB} \cong \overline{CD}$.

Diagram of Straightedge & Compass Construction #1A: Congruent Segments

Justification of Construction:
$\overline{AB}$ is a radius of circle A. $\overline{CD}$ is a radius of circle C. Since the compass opening was kept constant, the radii $\overline{AB}$ and $\overline{CD}$ are congruent. Therefore, $\overline{AB} \cong \overline{CD}$.

Construction #1B: Congruent Segments – Using *The Geometer's Sketchpad*
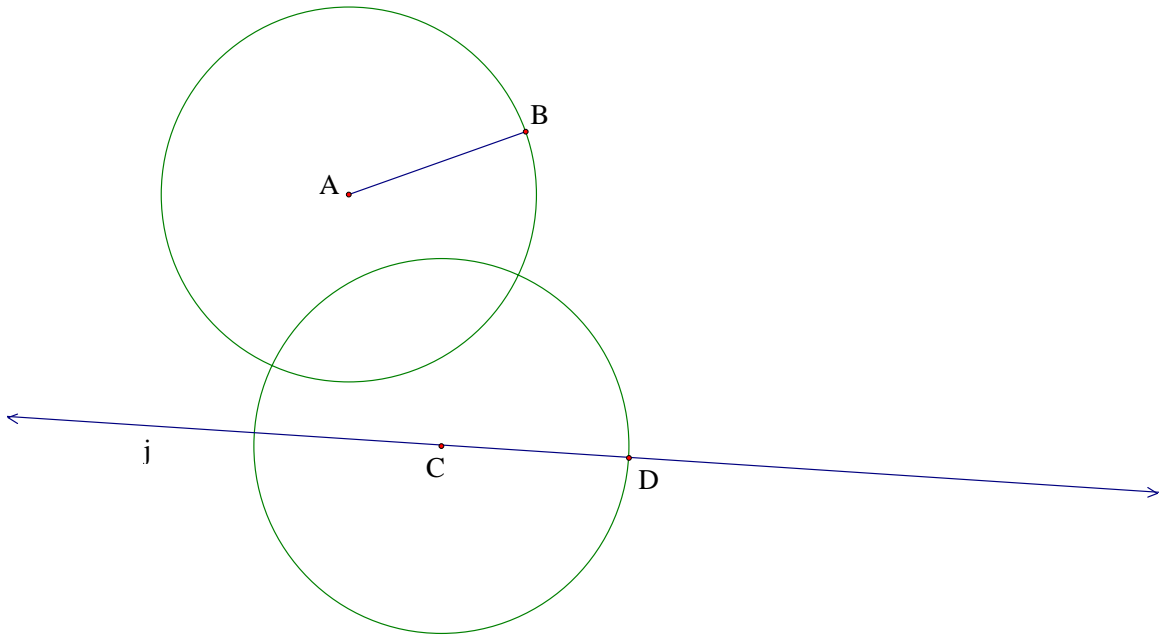
<u>Instructions:</u>

1. Click on the segment tool and draw any segment $\overline{AB}$. (The purpose of the construction is now to construct a segment congruent to $\overline{AB}$.)

2. Click on the line tool, draw a line and label it *j*.

3. Two points should have appeared on line *j* (since two points determine a line). Hide one of the points, and label the other point *C*.

4. Construct circle *A* with radius $\overline{AB}$.
   To do this: Select both point A and radius $\overline{AB}$. Then click on "Construct," then on "Circle by Center +Radius."

5. Translate circle A onto point C.
   To do this: Select points A and C (in that order). Then click on "Transform," then on "Mark Vector." Next, click on circle A (the curved part of the circle itself), then click on "Transform," then "Translate," (the translation vector box should be marked), then on "Translate."

6. Construct the (two) points of intersection of circle C and line *j*.
   To do this: Select both circle C (the circle itself – not the center) and line *j*. Then click on "Construct," then on "Intersections." Two intersection points should appear; hide one of the intersection points and label the other intersection point as *D*.

We have now completed our construction; $\overline{AB} \cong \overline{CD}$.

(Notice that you can change the length of the original segment $\overline{AB}$, as well as the position of line *j*, and the construction is still preserved.)

<u>Diagram of *Sketchpad* Construction #1B: Congruent Segments</u>

B

A

j

C

D

<u>Justification of Construction:</u>
$\overline{AB}$ is a radius of circle A. $\overline{CD}$ is a radius of circle C. $\odot A \cong \odot C$, since circle C was created by translating circle A. Since the two circles are congruent, their radii are congruent, therefore $\overline{AB} \cong \overline{CD}$.
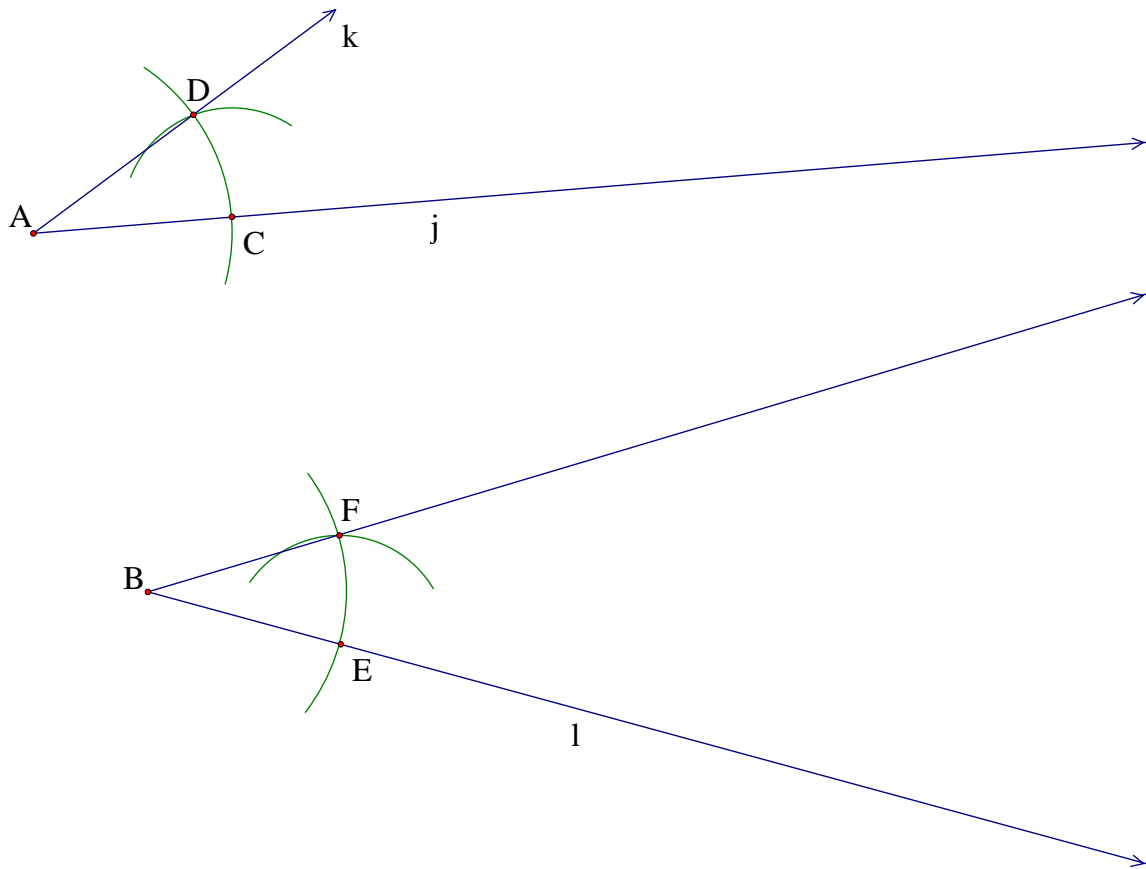
Construction #2A: Congruent Angles – Using a Straightedge and Compass

Instructions:

1. Use your straightedge to create an angle by drawing two rays with a common endpoint. Label this common endpoint (the vertex of the angle) as *A*. Label one of the rays as *j* and the other ray as *k*. (The purpose of the construction is to construct an angle congruent to angle *A*.)

2. Create the first side of the congruent angle by using your straightedge to draw another ray. Label the initial point of the ray as *B*, and label the ray as *l*.

3. Place the center of your compass on point *A* and draw an arc which passes through both ray *j* and ray *k*. Find the point of intersection of the arc and ray *j*, and label this point as *C*. Find the point of intersection of the arc and ray *k*, and label this point as *D*.

4. Keeping the compass open to the same radius, place the center of your compass on point *B* and draw an arc which intersects ray *l*. Label the point of intersection as *E*.

5. Place the center of your compass on point *C* and draw an arc which passes through point *D*.

6. Keeping the compass open to the same radius, place the center of your compass on point *E* and draw an arc which intersects the arc drawn in step 4 above. Label this point of intersection as *F*.

7. Use your straightedge to draw a ray which originates at point *B* and passes through point *F*.

We have now completed our construction; $\angle A \cong \angle B$. (Equivalently, if we wish to use three letters to name the angle, $\angle DAC \cong \angle FBE$.)

Diagram of Straightedge & Compass Construction #2A: Congruent Angles

k

D

A

C

j

F

B

E

l

Justification of Construction:

$\overline{AD}$ and $\overline{AC}$ are radii of circle A. $\overline{BF}$ and $\overline{BE}$ are radii of circle B. Since the compass opening was kept constant, the radii of circle A and circle B are congruent. Therefore, $\overline{AD} \cong \overline{BF}$ and $\overline{AC} \cong \overline{BE}$. Imagine drawing segments $\overline{CD}$ and $\overline{EF}$ (since a segment is determined by its two endpoints). Since the compass opening was kept constant, the radii of circle C and circle E are congruent; therefore $\overline{DC} \cong \overline{FE}$. We can then say that $\triangle ADC \cong \triangle BFE$ by SSS Congruence. Since the triangles are congruent, their corresponding parts are congruent, therefore $\angle A \cong \angle B$.

<u>Instructions:</u>

1. Create an angle by drawing two rays with a common endpoint.
   To do this: Click on the ray tool and then draw a ray. Two points should appear on the ray. Label the initial point of the ray as *A* and hide the other point. Use the text tool to label the ray as *j*. Then click on the ray tool and create another ray which also has *A* as its initial point. Hide the other point which appears on the second ray. Then click on the text tool and label the second ray as *k*. (The purpose of the construction is now to construct an angle congruent to angle *A*.)

2. Create the first side of the congruent angle by using the ray tool to draw another ray. Label the initial point of the ray as *B* and hide the other point on the ray.b Then click on the text tool and label this ray as *l*.

3. Click on the circle tool and draw a circle which has center *A*. Hide the point which appears on the circle itself.

4. Translate circle *A* onto point *B*.
   To do this: Select points *A* and *B* (in that order). Then click on "Transform," then on "Mark Vector." Next, click on circle *A* (the curved part of the circle itself), then click on "Transform," then "Translate," (the translation vector box should be marked), then on "Translate."

5. Create the point of intersection of ray *j* and circle *A*.
   To do this: Click on the selection tool (the arrow) and then select both ray *j* and circle *A* (the curved part of the circle – not center *A*). Then click on "Construct," then on "Intersection." A point of intersection should appear. Use the text tool to label this point as *C*.

6. Create the point of intersection of ray *k* and circle *A*.
   To do this: Click on the selection tool (the arrow) and then select both ray *k* and circle *A* (the curved part of the circle – not center *A*). Then click on "Construct," then on "Intersection." A point of intersection should appear. Use the text tool to label this point as *D*.

7. Create the point of intersection of ray *l* and circle *B*.
   To do this: Click on the selection tool (the arrow) and then select both ray *l* and circle *B* (the curved part of the circle – not center *B*). Then click on "Construct," then on "Intersection." A point of intersection should appear. Use the text tool to label this point as *E*.

8. Create a circle with center *C* which passes through point *D*.

To do this: Click on the selection tool (the arrow) and then select both point *C* and point *D* (in that order). Then click on "Construct," and then on "Circle By Center+Point."

9. Translate circle *C* onto point *E*.
    To do this: Select points *C* and *E* (in that order). Then click on "Transform," then on "Mark Vector." Next, click on circle *C* (the curved part of the circle itself), then click on "Transform," then "Translate," (the translation vector box should be marked), then on "Translate."

10. Construct the (two) points of intersection of circle *B* and circle *E*.
    To do this: Click on the selection tool (the arrow) and select both circle *B* and circle *E* (the circles themselves – not their center points). Then click on "Construct," then on "Intersections." Two intersection points should appear; hide one of the intersection points and label the other intersection point as *F*.
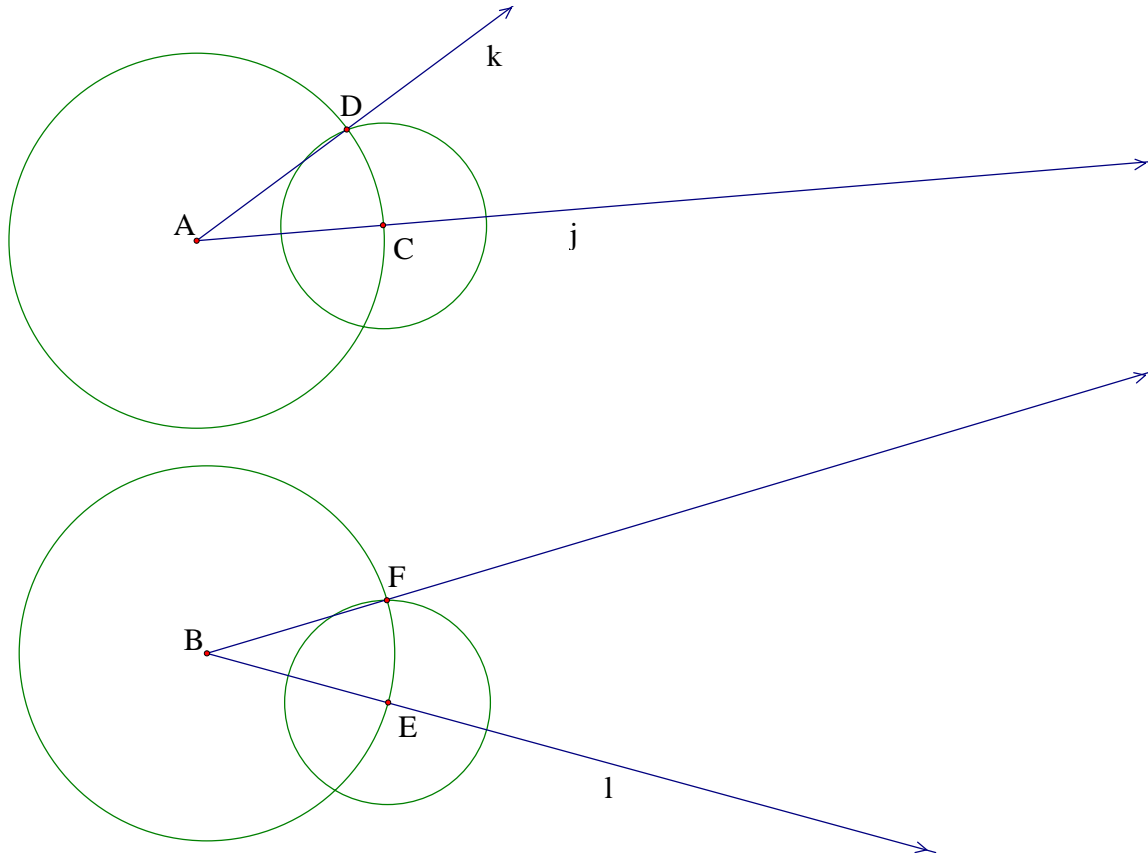
11. Draw a ray which originates at point *B* and passes through point *F*.
    To do this: Click on the selection tool (the arrow) and select both point *B* and point *F* (in that order). Then click on "Construct," and then on "Ray."

We have now completed our construction; $\angle A \cong \angle B$. (Equivalently, if we wish to use three letters to name the angle, $\angle DAC \cong \angle FBE$.)

(Notice that you can change the original angle by dragging around point *A*, ray *j* or ray *k*, and the construction is still preserved.)

<u>Diagram of *Sketchpad* Construction #2B: Congruent Angles</u>

<u>Justification of Construction:</u>

$\overline{AD}$ and $\overline{AC}$ are radii of circle *A*. $\overline{BF}$ and $\overline{BE}$ are radii of circle *B*. $\odot A \cong \odot B$, since circle *B* was created by translating circle *A*. Since the two circles are congruent, their radii are congruent, therefore $\overline{AD} \cong \overline{BF}$ and $\overline{AC} \cong \overline{BE}$. Imagine drawing segments $\overline{CD}$ and $\overline{EF}$ (since a segment is determined by its two endpoints). $\odot C \cong \odot E$, since circle *E* was created by translating circle *C*. Since these two circles are congruent, their radii are congruent, therefore $\overline{DC} \cong \overline{FE}$. We can then say that $\triangle ADC \cong \triangle BFE$ by SSS Congruence. Since the triangles are congruent, their corresponding parts are congruent, therefore $\angle A \cong \angle B$.

Construction #3A: Parallel Lines – Using a Straightedge and Compass
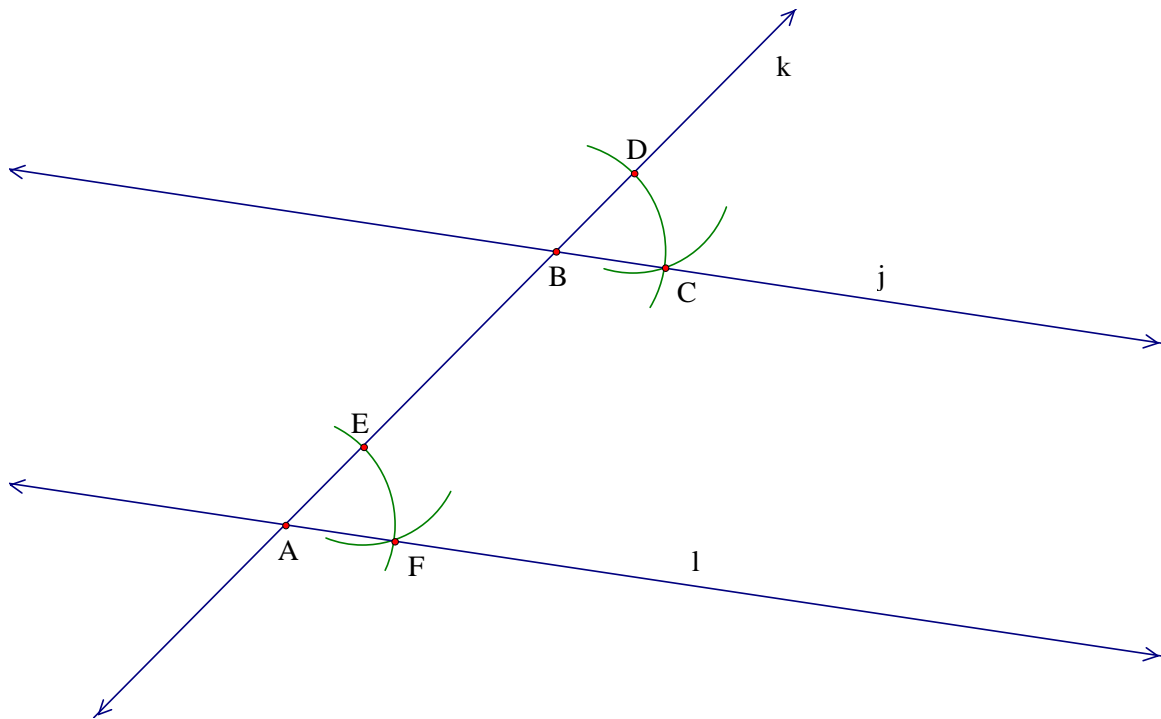
Instructions:

1. Use your straightedge to draw any line and label it as *j*

2. Draw a point which is not on line *j*. Label this point as *A*. .(The purpose of this construction is now to construct a line parallel to line *j* which passes through point *A*..)

3. Place a point *B* on line *j*.

4. Use your straightedge to draw a line which passes through points *A* and *B*. Then label the line as *k*. (Note: Line *k* is known as a transversal.)

Notice that the two intersecting lines have formed four angles at point *B*. Choose one of those angles; in our diagram, we will choose the angle to the upper right of point *B*. (If you have a similar diagram and you choose the same angle, it will be easier for you to refer to our diagram of this construction.) We now wish to draw a corresponding angle at point *A* which is congruent to this chosen angle. The steps below are based on those from Construction #2: Congruent Angles, but the instructions have been changed to reflect the properties of our particular diagram.

5. Place the center of your compass on point *B* and draw an arc which passes through both line *j* and line *k*. (We have chosen to draw an arc to the upper right of point *B*, as indicated above.) Find the point of intersection of the arc and line *j*, and label this point as *C*. Find the point of intersection of the arc and line *k*, and label this point as *D*.

6. Keeping the compass open to the same radius, place the center of your compass on point *A* and draw an arc which intersects line *l*. (We have chosen to draw the arc to the upper right of point *A,* to create a corresponding angle to the one at point *B*.) Label the point of intersection as *E*.

7. Place the center of your compass on point *D* and draw an arc which passes through point *C*.

8. Keeping the compass open to the same radius, place the center of your compass on point *E* and draw an arc which intersects the arc drawn in step 6 above. Label this point of intersection as *F*.

9. Use your straightedge to draw a line which originates at point *A* and passes through point *F*. Label this line as *l*.

We have now completed our construction; line *j* is parallel to line *l*. (i.e. *j // l* .)

Diagram of Straightedge & Compass Construction #3A: Parallel Lines

k

D

B   j

C

E

A   l

F

Justification of Construction:
$\overline{BD}$ and $\overline{BC}$ are radii of circle B. $\overline{AE}$ and $\overline{AF}$ are radii of circle A. Since the compass opening was kept constant, the radii of circle A and circle B are congruent. Therefore $\overline{BD} \cong \overline{AE}$ and $\overline{BC} \cong \overline{AF}$. Imagine drawing segments $\overline{DC}$ and $\overline{EF}$ (since a segment is determined by its two endpoints). Since the compass opening was kept constant, the radii of circle D and circle E are congruent; therefore $\overline{DC} \cong \overline{EF}$. We can then say that $\triangle BDC \cong \triangle AEF$ by SSS Congruence. Since the triangles are congruent, their corresponding parts are congruent, therefore $\angle DBC \cong \angle EAF$. We now refer to the following theorem: If two lines are cut by a transversal and their corresponding angles are congruent, then the lines are parallel. If we identify line k as the transversal of lines j and l, we can see that $\angle DBC$ and $\angle EAF$ are corresponding angles. Since $\angle DBC \cong \angle EAF$, we conclude that line j is parallel to line l (i.e. $j // l$).

Construction #3B: Parallel Lines – Using *The Geometer's Sketchpad*

<u>Instructions:</u>

1. Click on the line tool and draw a line. Two points should appear on the line; hide both of these points and then label the line as *j*.

2. Click on the point tool and draw a point which is not on line *j*. Label this point as *A*. (The purpose of this construction is now to construct a line parallel to line *j* which passes through point A.)

3. Construct point *B* on line *j*.
    To do this: Click on the selection tool (the arrow) and select line *j*. Then click on "Construct," then "Point On Line." A point should appear; label this point as *B*. (Note: You may use the selection tool to drag point *B* to any position on line *j*.)

4. Draw a line which passes through points *A* and *B*.
    To do this: Click on the selection tool (the arrow) and select both points *A* and *B*. Then click on "Construct," then "Line." Use the text tool to label this line as *k*. (Note: Line *k* is known as a transversal.)

Notice that the two intersecting lines have formed four angles at point *B*. Choose one of those angles; in our diagram, we will choose the angle to the upper right of point *B*. (If you have a similar diagram and you choose the same angle, it will be easier for you to refer to our diagram of this construction.) We now wish to draw a corresponding angle at point *A* which is congruent to this chosen angle. The steps below are based on those from Construction #2: Congruent Angles, but the instructions have been changed to reflect the properties of our particular diagram.
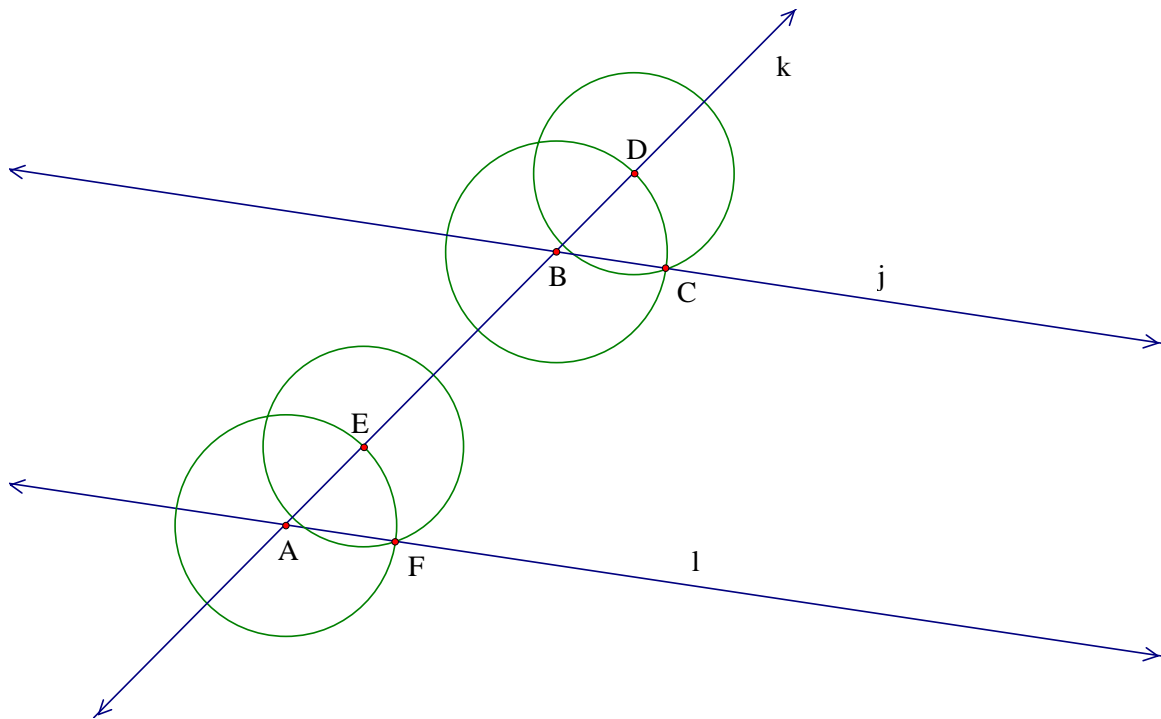
5. Click on the circle tool and draw a circle which has center *B*. Hide the point which appears on the circle itself.

6. Translate circle *B* onto point *A*.
    To do this: Select points *B* and *A* (in that order). Then click on "Transform," then on "Mark Vector." Next, click on circle *B* (the curved part of the circle itself), then click on "Transform," then "Translate," (the translation vector box should be marked), then on "Translate."

7. Create the points of intersection of line *j* and circle *B*.
    To do this: Click on the selection tool (the arrow) and then select both line *j* and circle *B* (the curved part of the circle – not center *B*). Then click on "Construct," then on "Intersections." Two points of intersection should appear. Hide one of the points (we choose to hide the point on the left, since we are focusing on the angle to the upper right of point *B*) and use the text tool to label the other intersection point as *C*.

8. Create the points of intersection of line *k* and circle *B*.
   To do this: Click on the selection tool (the arrow) and then select both line *k* and circle *B* (the curved part of the circle – not center *B*). Then click on "Construct," then on "Intersections." Two points of intersection should appear. Hide one of the points (we choose to hide the lower point, since we are focusing on the angle to the upper right of point *B*) and use the text tool to label the other intersection point as *D*.

9. Create the points of intersection of line *k* and circle *A*.
   To do this: Click on the selection tool (the arrow) and then select both line *k* and circle *A* (the curved part of the circle – not center *A*). Then click on "Construct," then on "Intersections." Two points of intersection should appear. Hide one of the points (we choose to hide the lower point, since we want to create a corresponding angle to the upper right of point *A*) and use the text tool to label the other intersection point as *E*.

10. Create a circle with center *D* which passes through point *C*.
    To do this: Click on the selection tool (the arrow) and then select both point *D* and point *C* (in that order). Then click on "Construct," and then on "Circle By Center+Point."

11. Translate circle *D* onto point *E*.
    To do this: Select points *D* and *E* (in that order). Then click on "Transform," then on "Mark Vector." Next, click on circle *D* (the curved part of the circle itself), then click on "Transform," then "Translate," (the translation vector box should be marked), then on "Translate."

12. Construct the (two) points of intersection of circle *A* and circle *E*.
    To do this: Click on the selection tool (the arrow) and select both circle *A* and circle *E* (the circles themselves – not their center points). Then click on "Construct," then on "Intersections." Two intersection points should appear; hide one of the intersection points (we choose to hide the point at the left, since we are focusing on the angle to the upper right of point *A*) and use the text tool to label the other intersection point as *F*.

13. Draw a line that passes through points *A* and *F*.
    To do this: Click on the selection tool (the arrow) and select both point *A* and point *F*. Then click on "Construct," and then on "Line." Label this line as *l*.

We have now completed our construction; line *j* is parallel to line *l*. (i.e. *j* // *l* .)

(Notice that you can change the diagram by dragging around point *A*, point *B*, line *j* or line *k*, and the construction is still preserved.)

Diagram of *Sketchpad* Construction #3B: Parallel Lines

Justification of Construction:

$\overline{BD}$ and $\overline{BC}$ are radii of circle B. $\overline{AE}$ and $\overline{AF}$ are radii of circle A. $\odot A \cong \odot B$, since circle A was created by translating circle B. Since the two circles are congruent, their radii are congruent, therefore $\overline{BD} \cong \overline{AE}$ and $\overline{BC} \cong \overline{AF}$. Imagine drawing segments $\overline{DC}$ and $\overline{EF}$ (since a segment is determined by its two endpoints). $\odot D \cong \odot E$, since circle E was created by translating circle D. Since these two circles are congruent, their radii are congruent, therefore $\overline{DC} \cong \overline{EF}$. We can then say that $\triangle BDC \cong \triangle AEF$ by SSS Congruence. Since the triangles are congruent, their corresponding parts are congruent, therefore $\angle DBC \cong \angle EAF$. We now refer to the following theorem: If two lines are cut by a transversal and their corresponding angles are congruent, then the lines are parallel. If we identify line k as the transversal of lines j and l, we can see that $\angle DBC$ and $\angle EAF$ are corresponding angles. Since $\angle DBC \cong \angle EAF$, we conclude that line j is parallel to line l (i.e. $j \,/\!/\, l$).