Patterns by Design

Purpose:
Participants will use a set of Tangrams to make designs for a quilting square that are similar triangles. They will compare corresponding angles and sides of the different triangles and make conjectures.

Overview:
Participants will make a set of triangles with Tangram pieces and compare corresponding angles and sides. They will make conjectures about attributes of similar triangles and explore relationships among corresponding sides, corresponding perimeters, and corresponding areas.

TExES Mathematics 4-8 Competencies. The beginning teacher:

III.008.A Selects and uses appropriate units of measurement (e.g., temperature, money, mass, weight, area, capacity, density, percents, speed, acceleration) to quantify, compare, and communicate information.
III.008.E Applies the Pythagorean theorem, proportional reasoning, and right triangle trigonometry to solve measurement problems.
III.009.A Understands concepts and properties of points, lines, planes, angles, length, and distances.
III.009.B Analyzes and applies the properties of parallel and perpendicular lines.
III.009.C Uses the properties of congruent triangles to explore geometric relationships and prove theorems.
III.010.A Uses and understands the development of formulas to find lengths, perimeters, areas, and volumes of basic geometric figures.
III.010.B Applies relationships among similar figures, scale, and proportion and analyzes how changes in scale affect area and volume measurements.
III.010.C Uses a variety of representations (e.g., numeric, verbal, graphic, symbolic) to analyze and solve problems involving two-and three-dimensional figures such as circles, triangles, polygons, cylinders, prisms, and spheres.

TEKS Mathematics Objectives. The student is expected to:

4.8A Identify right, acute, and obtuse angles.
4.8B Identify models of parallel and perpendicular lines.
4.8C Describe shapes and solids in terms of vertices, edges, and faces.
4.12 Measure to solve problems involving length, including perimeter, time, temperature, and area.
5.2B Compare two fractional quantities in problem-solving situations using a variety of methods, including common denominators.
5.7A Identify critical attributes including parallel, perpendicular, and congruent parts of geometric shapes and solids.
5.7B Use critical attributes to define geometric shapes or solids.
6.2C Use multiplication and division of whole numbers to solve problems including situations involving equivalent ratios and rates.
6.3A Use ratios to describe proportional relationships.
6.3B Represent ratios and percents with concrete models, fractions, and decimals.
6.6A Use angle measurements to classify angles as acute, obtuse, or right.
6.6B Identify relationships involving angles in triangles and quadrilaterals.
7.3B Estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.
7.6B Use properties to classify shapes including triangles, quadrilaterals, pentagons, and circles.
7.6D Use critical attributes to define similarity.
8.2D Use multiplication by a constant factor (unit rate) to represent proportional relationships.

8.6A Generate similar shapes using dilations including enlargements and reductions.
8.10A Describe the resulting effects on perimeter and area when dimensions of a shape are changed proportionally.

Terms.
Perimeter, area, triangle, right triangle, acute angle, complementary angles, square units, units, square, parallelogram, ratio, proportion, congruent, similar, corresponding angles, corresponding sides, length, Tangrams, parallel, perpendicular

Materials.
- Tangrams (1 set per two participants)
- Paper for recording and cutting strips of paper
- Map pencils (optional)
- Straight edge
- Patty paper
- Scissors
- Cellophane tape

Transparencies.
- *Patterns by Design*

Activity Sheet(s).
- *Patterns by Design*

Procedure.

Steps	Questions/Math Notes
1. Have participants read the problem *Patterns by Design* the day before the problem is to be discussed in groups. Provide some time for exploration in pairs with the Tangrams. 2. Have a discussion about the relationships among the different Tangram polygons before assigning the problem. 3. Ask participants to work individually or in pairs on the triangle designs made with Tangrams so that there is ample time for exploration as some of these designs may require more time. 4. Ask them to record the different triangle designs and bring their drawings to class the next day for discussion. 5. Participants may use a set of Tangrams or the template provided on the Activity Sheet *Patterns by Design.*	Ask participants questions about the relationships among the Tangram polygons observed. *How does the small triangle relate to the other Tangram polygons?* *When the 7 Tangram polygons are assembled to form a large square with an area of 1 square unit, how can you find the area of each of the 7 Tangram polygons?* *How do the perimeters of the Tangram polygons compare?* *What are some properties of the small, medium, and large triangles?* *If the length of a leg of the small triangle represents 1 unit, how can you find the perimeter of the original Tangram square made with all 7 pieces?*
6. The next day, have participants work in small groups of 2 to 4 and discuss their answers to Destiny's questions. 7. Ask several groups with different solutions to write their solution on overhead transparencies. 8. Have a whole group discussion of the problem and different solutions. 9. Have participants work on the extension with pattern blocks to investigate properties of similar polygons.	*What similarities did you observe among the triangles in the different sets? Differences?* *How would you classify the triangles in the sets?* *How did you determine the ratio of corresponding sides for two different-sized triangles? Corresponding angles?* *Are the triangles in the different sets similar? How do you know?* *What can you generalize about similar triangles?* *What information is sufficient to show two triangles are similar?* *Would this be enough information to show two squares are similar? Two rectangles? Explain your reasoning.* *How did you determine the ratio of corresponding perimeters of two similar triangles?* *How does this ratio compare with the ratio of the corresponding sides of the similar triangles?*

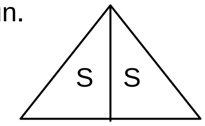
Steps	Questions/Math Notes
	If the ratio of two corresponding sides of similar triangles is a:b, then what is the ratio of the corresponding perimeters of these similar triangles? How do you know? *What did you observe about the ratio of corresponding sides of two similar triangles and the ratio of their corresponding areas?* *What do you predict the ratio of corresponding altitudes to the hypotenuse of two right isosceles triangles will be? How could you verify this?* *What do you think the ratio of corresponding parts of two similar triangles will be? Explain your reasoning.*

Solution:

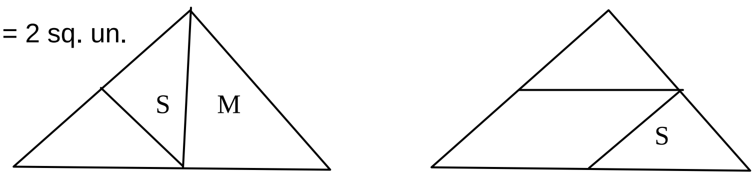
1. There are five different sets of triangles that can be formed with 1 set of Tangrams. More than one triangle can be formed for each set except for the one made up of two small triangles. Possible arrangements for each different set of triangles are shown below and are not drawn to scale.

Note: The length of a leg of a small triangle represents 1 unit .

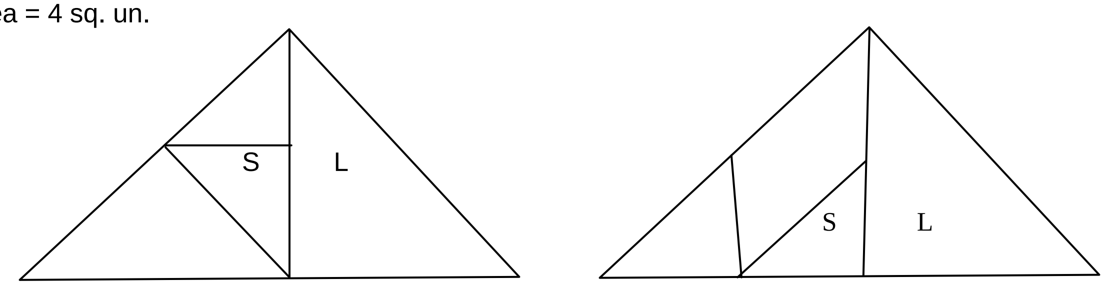
Set 1: Area = 1 sq. un.



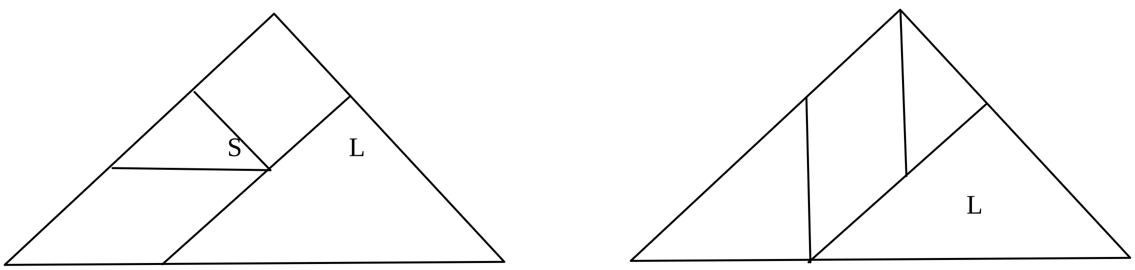
Set 2: Area = 2 sq. un.



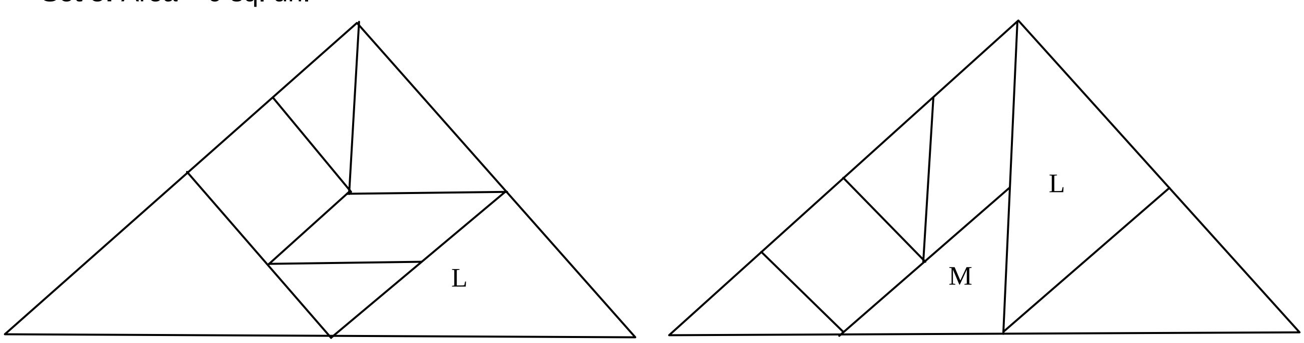
Set 3: Area = 4 sq. un.



Set 4: Area = 4.5 sq. un.



Set 5: Area = 9 sq. un.



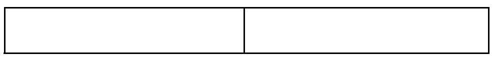
Solution continued:

2. Patty paper can be used to trace the angles of any triangle in one set and compared to corresponding angles of any triangle in another set. The corresponding angles will be congruent. Each right triangle formed by Tangrams is a right isosceles triangle with complementary acute angles. Each acute angle has a measure of 45 degrees.

   Set 1 and Set 3 Comparisons:
   The ratio of corresponding sides is 1:2. This can be verified by cutting strips of paper about 1 centimeter wide to represent the length of each side of a triangle in Set 1. Compare the length of a strip representing the length of a side of the smaller triangle (Set 1) to the length of a strip representing a corresponding side of the larger triangle (Set 3) as shown below. Repeat this for the remaining sides of the two triangles.

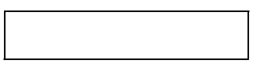
   

   (Length of a corresponding side of a triangle in Set 1)

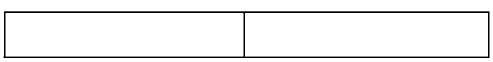
   

   (Length of a side of a triangle in Set 3 folded in half)

3. Cut strips of paper (about 1 cm wide) for the lengths of the sides of a triangle in Set 1; repeat for a triangle in Set 3. Tape the strips for each triangle end-to-end to represent the lengths of three sides of each triangle. Fold the longer strip (perimeter of triangle in Set 3) in half and compare each half to the length of the shorter strip (perimeter of triangle in Set 1). This comparison shows that the ratio of the perimeters of the two triangles is 1:2 as shown below.

   (perimeter of triangle in Set 1)

   (perimeter of triangle in Set 3)

   Perimeter of △ in Set 1: Perimeter of △ in Set 3 = 1: 2

4. Each small triangle has an area of ½ sq. unit using the length of a leg as 1 unit. A medium triangle has an area of 1 sq. unit. A triangle in Set 3 has an area of 4 sq. units. This can be verified by tracing a medium triangle on patty paper four times and cutting out the four medium triangles. By placing these four medium triangles on a triangle in Set 3, the area of the triangle in Set 3 can be shown to be 4 times the area of a medium triangle. This leads to the following ratio of their areas:
   Area of △ in Set 1: Area of △ in Set 3 = 1: 4

   It can also be shown that this ratio of corresponding areas is the same as the ratio of the squares of the corresponding sides of the triangles in Set 1 and Set 3.

   Using the same procedure with paper strips, ratios of corresponding sides and corresponding perimeters can be verified for other sets of triangles. Some estimation is necessary as in the case of triangles in Sets 2 and 3. The ratio of corresponding sides is about 1: 1 2/5 or 1: 7/5 = 5/7. When this ratio is squared, the ratio of corresponding areas is 25/49 or about ½.

Extension:

1. Use the orange squares, blue rhombi, and red trapezoids from a set of pattern blocks to investigate similarity among different kinds of quadrilaterals. Build a set of three squares starting with a square one unit on each side. Build the next larger square with two units on each side; three units on each side. Repeat this process with the blue rhombi and the red trapezoids.
2. Record results in a table like the one shown below.
3. Make generalization(s) about similar squares, rhombi, and trapezoids.
4. Use the information in the table to make generalizations about the relationship between corresponding sides and corresponding perimeters; between corresponding sides and corresponding areas.

Pattern block	Sketch of figure with side of 2 un.	Sketch of figure with side of 3 un.	Ratio of corresponding sides	Ratio of corresponding perimeters	Ratio of corresponding areas
Orange Square					
Blue Rhombus					
Red Trapezoid					

Answers:

1. Refer to the table.

2. The results in the table above show that the ratio of corresponding sides is 2:3; the ratio of corresponding perimeters is 2:3; the ratio of corresponding areas is 4:9.

3. The ratio of corresponding sides of similar squares, rhombi, and trapezoids is the same as the ratio of their corresponding perimeters. All squares are similar but not all rectangles are similar. All rhombi are not always similar. Isosceles trapezoids are not always similar.

4. In similar figures having a ratio of corresponding sides *a : b* , the ratio of corresponding perimeters is also *a : b.* The ratio of corresponding areas is $a^2 : b^2$.

Reference:

The Super Source, Geometry 7^{th}-8^{th} :*Sal's Similar Sails.* Cuisenaire Company of America, Inc.