# Tantalizing Tessellations

**Purpose:**
Participants will make tessellations in a plane using transformations to create designs. They will describe how the properties of a transformation were used in making and tessellating the design.

**Objective:**
Participants will use 3”x5” plain index cards, scissors, and tape to create designs named tessellations using transformations such as reflections, glide-reflections, translations, and rotations in the plane. They will use markers or colored pens/pencils to personalize the designs for display. Then participants will view another group’s design and write an explanation of how to create it using transformations.

**TExES Mathematics 4-8 Competencies.** The beginning teacher:
III.011.B Uses translations, reflections, glide-reflections, and rotations to demonstrate congruence and to explore the symmetries of figures.
III.011.D Uses symmetry to describe tessellations and shows how they can be used to illustrate geometric concepts, properties, and relationships.
V.016.B Uses mathematics to model and solve problems in other disciplines, such as art, music, science, social science, and business.
VI.018.H Understands how technological tools and manipulatives can be used appropriately to assist students in developing, comprehending, and applying mathematical concepts.

**TEKS Mathematics Objectives.** The student is expected to:
4.9.A Demonstrate translations, reflections, and rotations to verify that two shapes are congruent.
4.9.B Use translations, reflections, and rotations to verify that two shapes are congruent.
4.9.C Use reflections to verify that a shape has symmetry.
5.8.A Sketch the results of translations, rotations, and reflections.
6.11.A Identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics.
7.13.A Identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics.
8.14.A Identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics.

**Terms.**
Reflection, translation, glide-reflection, rotation, transformation, isometry, symmetry, congruent, tessellation

**Materials.**
- 3x5 plain index cards ( 1 set per 2 tables)
- Scissors
- Cellophane tape
- Colored markers, map pencils, and/or crayons
- Poster paper for display and mounting of designs
- Pattern blocks
- Patty paper or tracing paper
- 81/2”x11” paper

**Transparencies.**
- *Tantalizing Tessellations*

**Activity Sheet(s).**

- *Activity 1: Tantalizing Tessellations, Exploring With Pattern Blocks and Templates*
- *Activity 2: Tantalizing Tessellations, Using Index Cards*

**References:**


http://library.thinkquest.org/16661/escher.html

http://www.nga.gov/collection/gallery/ggescher-main1.html

http//fc.nbsc.org/~5/math99/Tessellations.htm

Serra, Michael(1997). *Discovering Geometry, An Inductive Approach.* Berkeley, CA: Key Curriculum Press.

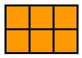

**Procedure:**

| Steps | Questions/Math Notes |
|---|---|
| 1. Have participants read about *Tantalizing Tessellations* from the transparency on the overhead.<br><br>Ask them to share their understanding about tessellations by the famous artist, M.C. Escher.<br><br>If possible, show one of the Escher designs on the overhead or use a poster of an Escher design. | Ask participants questions to clarify and extend their thinking about transformations and how they can be used to create designs to tessellate a plane.<br><br>*What can you share about the tessellations created by M.C. Escher?*<br><br>*What do you observe about an Escher design?* |
| 2. Have participants begin with Activity 1: *Tantalizing Tessellations, Exploring with Pattern Blocks and Templates.* Have them explore tiling the plane with a set of pattern blocks beginning with the simplest pattern using the orange square.<br><br>Explain that tiling the plane means that they must cover the plane with the pattern block or pattern block pattern so that there are no holes or spaces between the pattern blocks. Also make the connection between tiling the plane and tessellations which are the repeated patterns that tile the plane. | *When you look at a tile floor with square tiles, what do you observe about the tiles?*<br><br>*What do most of the pattern blocks have in common?*<br><br>*Why do you think it is possible to tile the plane with that particular pattern block?*<br><br>*What patterns seem to emerge as you tile the plane with other pattern blocks? Explain.*<br><br>*Do you think you could tile the plane with any regular polygon? An irregular polygon? A combination of patterns blocks that form a design? Explain.* |
| 3.Debrief the activity *Tantalizing Tessellations, Exploring with Pattern Blocks and Templates.* | *Which pattern blocks tiled the plane? Why do you think this was possible?*<br><br>*Were you able to tile the plane with every pattern block? Explain.*<br><br>*What did you discover about tiling the plane with a regular pentagon? A regular octagon?* |

| | |
|---|---|
| | *What did you discover about tiling the plane with an irregular pentagon?*<br><br>*What seems to be a generalization for tiling the plane with regular polygons? Explain.*<br><br>*What combination of pattern blocks did you use to create a mosaic design for tiling the plane? Please share what you did.*<br><br>*Why does this combination of pattern blocks make it possible to tile the plane?*<br><br>*How can the use of color help you create other mosaic designs?* |
| 4. Have participants begin Activity 2 *Tantalizing Tessellations, Using Index Cards.* They are to follow the procedure outlined and answer the questions asked.<br><br>Use an index card, tape, and a pair of scissors to demonstrate what they are to do as outlined on Activity 2 in steps 1-5.<br><br>Inform participants that they are to record the procedure that they used to create their design and follow steps 6-12 of Activity Sheet 2 noting the transformations used. | *What other transformation(s) could have been used?*<br><br>*How would the use of the transformation(s)t affect the design?*<br><br>*What interesting subject do you observe in the design created?*<br><br>*What is another subject that you observe?*<br><br>*How could the use of color affect the resulting tessellation?* |
| 5. As participants complete their designs using tessellations, have them post their work around the room with the procedure used taped to the back.<br><br>Have groups go to a partner group's work to analyze the work. Each group is to describe a procedure that could be used to create the design that was tessellated.<br><br>After analyzing the partner group's work, groups are to return to their tables and use the procedure they determined to replicate the design. | *Is there another way that you could have used your design to create a different tessellation in the plane? Explain.*<br><br>*What made you think to use that transformation?*<br><br>*How would your design differ if you used a different transformation?* |
| 6. Debrief the activity with the whole group asking questions to connect their learning about tessellations using designs vs. polygons. | *How did analyzing another group's work extend your thinking about tessellations?*<br><br>*Did any group(s) use a combination of transformations vs. one transformation at a time? Demonstrate the process in the group's work.*<br><br>*What are some strategies that you would use in creating your next design based upon your observation(s) of other designs? Explain.* |

**Possible solutions:**

**Activity Sheet 1:** *Tantalizing Tessellations, Exploring With Pattern Blocks and Templates*

1.  This tiling can be done by starting with one orange square and translating the square in the plane.

2. The tiling of the plane using each of the other pattern blocks will be done in a similar manner as #1.

3. All of the pattern blocks can be used to tile a floor without any spaces or holes.

4. A regular pentagon cannot be used to tile the plane without spaces or holes. When the regular pentagons (3) are arranged about a common vertex, the sum of the interior angles is 324°. The sum of the interior angles needs to be 360° to close the gaps.

   Total number of degrees of the interior angles: (n-2) (180) where n represents the number of sides of the polygon
   (5-2)(180) = 3(180) = **540°**

   Number of degrees in one interior angle: (n-2)(180) / n
   540 / 5
   **108°**

   Sum of the interior angles of three regular pentagons sharing a common vertex:3(108)= **324°**

5. A regular octagon will not tessellate or tile the plane as the sum of the interior angles of the maximum number (2) that can share a common vertex is 270°. There would be spaces in the tiling. The following formulas were used to determine this.

   Total number of degrees of the interior angles: (n-2) (180) where n represents the number of sides of the polygon
   (8-2) (180) = 6(180) = **1080°**

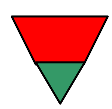
   Number of degrees in one interior angle: (n-2)(180) / n
   1080 / 8
   **135°**

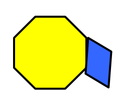
   Sum of the interior angles of two regular octagons sharing a common vertex: 2(135°)=**270°**

6. Regular polygons with an interior angle having a measure that is a multiple of 360° can tessellate or tile the plane.

7. On page 403 of *Discovering Geometry, An Inductive Approach* by Michael Serra, there is a discussion about irregular pentagon tessellations that have been discovered. There is still an unanswered question about whether there are still more to be discovered.

8. Answers will vary. Examples of combinations:





**Activity Sheet 2:** *Tantalizing Tessellations, Using Index Cards*

1- 12 Answers will vary according to the design.

**Extension:** ***Analyze This!***

Select one of Escher's works and analyze the transformations that he may have used to create the design to tessellate the plane.

**Possible solution:** Refer to Michael Serra's *Discovering Geometry, An Inductive Approach,* pp. 407-408, pp. 413-415, and pp. 422-423 for examples.