# Look Alikes

**Purpose:**
Participants will investigate ways to construct congruent triangles given three measurements (sides and/or angles) and validate the SSS, SAS, and ASA congruence postulates using technology.

**Objective:**
Participants will use the TI-83+ graphing calculator with Cabri Junior™ software to investigate conditions for constructing a triangle congruent to a given triangle (SSS,SAS, and ASA). They will validate the constructions using the MEASURE option on the F5 menu and supporting geometric definitions, axioms/postulates, and/or theorems.

**TExES Mathematics 4-8 Competencies.** The beginning teacher:
III.009.A Understands concepts and properties of points, lines, planes, angles, lengths, and distances.
III.009.D Describes and justifies geometric constructions made using compass and straight edge and other appropriate technologies.
V.016.E Demonstrates an understanding of the use of visual media such as graphs, tables, diagrams, and animations to communicate mathematical information.
VI.018.H Understands how technological tools and manipulatives can be used appropriately to assist students in developing, comprehending, and applying mathematical concepts.

**TEKS Mathematics Objectives.** The student is expected to:
4.9.B Use translations, reflections, and rotations to verify that two shapes are congruent.
4.12 Measure to solve problems involving length, including perimeter, time, temperature, and area.
4.15.A Explain and record observations using objects, words, pictures, numbers, and technology.
5.7.A Identify critical attributes including parallel, perpendicular, and congruent parts of geometric shapes and solids.
5.11.A Measure to solve problems involving length (including perimeter), weight, capacity, time, temperature, and area.
5.15.A Explain and record observations using objects, words, pictures, numbers, and technology.
6.8.C Measure angles.
6.13.B Validate his/her conclusions using mathematical properties and relationships.
7.15.B Validate his/her conclusions using mathematical properties and relationships.
8.16.B Validate his/her conclusions using mathematical properties and relationships.

**Terms.**
Congruent, triangle, angle, side, measure, length, construct, circle, compass, segment, line, point, postulate/axiom, corresponding

**Materials.**
- TI-83+ graphing calculators with Cabri Junior™ software
- Compass and straight edge (optional)

**Transparencies.**
- *Look Alikes*

**Activity Sheet(s).**
- *Look Alikes*

**Procedure:**

| Steps | Questions/Math Notes |
|---|---|
| 1. Have participants read the introduction to the investigations with the TI-83+ graphing calculator and Cabri Junior™ software using Transparency *Look Alikes* .<br><br>Each participant should have his/her own graphing calculator to do these investigations. Participants should work in small groups to discuss their results.<br><br>Ask participants to smile if they have some experience in using the software and have them sit among the different groups to help others with the use of the software as needed.<br><br>If participants have not had previous experience in constructing a segment congruent to a given segment or an angle congruent to a given angle using compass and straight edge, a quick review may be necessary. The constructions with Cabri Junior™ will be similar.<br><br>Monitor the work of participants and assist them with the use of the graphing calculator and Cabri Junior™ | Ask participants questions to clarify and extend their understanding of congruent polygons and their corresponding parts.<br><br>*What does "included angle" mean in the SAS congruence postulate? "included side" in the ASA congruence postulate?*<br><br>*How do you construct a segment congruent to a given segment with a compass and straight edge?*<br><br>*How can you validate your construction using definitions, properties, postulates and/or theorems?*<br><br>*How do you construct an angle congruent to a given angle using a compass and straight edge?*<br><br>*How could you verify the construction?* |
| 2. Ask for volunteers to share their findings on the overhead using an overhead graphing calculator.<br><br>Have them discuss how they verified their constructions using definitions, properties, postulates and/or theorems from geometry. | *How did you verify that the triangle constructed is congruent to the given triangle?*<br><br>*Could you have constructed this triangle another way? Explain.*<br><br>*Could you use AAA to construct a triangle congruent to a given triangle? Why or why not?*<br><br>*How could the use of the graphing calculator with Cabri Junior™ software help students understand the triangle congruence postulates?* |

**Solution:**

1. Construction of $\Delta DEF$ congruent to $\Delta ABC$ given three sides (SSS) using the TI-83+ graphing calculator with Cabri Junior™ software:
   - Press F2 and select **Triangle** followed by ENTER. Draw a scalene triangle and label its vertices with the letters A, B, and C (figure 1).
   - Press F2 and select **Line** followed by ENTER. Draw a line on the screen, locate point D on the line, and label the point with the letter D as shown in figure 2.

- Press F3 and select **Compass** followed by ENTER. Move the cursor to point B and press ENTER; then move it to point C and press ENTER. Move the center of circle B to point D and press ENTER. Now **BC=DE** because congruent circles have equal radii.

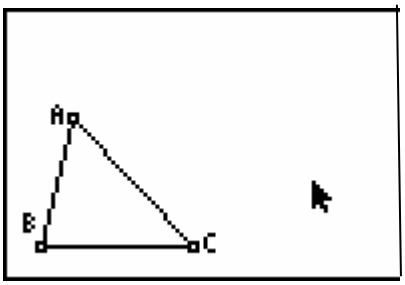

Figure 1

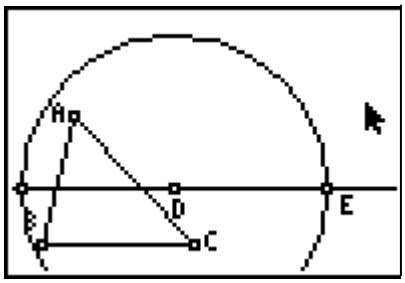

Figure 2

- Press F3 and select **Compass**. Construct circle D with radius AB. Construct circle E with radius AC and label the point of intersection of circles D and E with the letter F as shown in figure 3. Draw segments DF and EF as shown in figure 4. **DF = AB** and **EF = AC** because radii of congruent circles have equal measures.

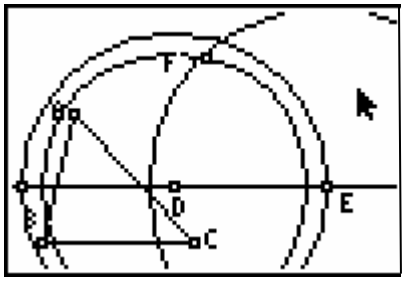

Figure 3

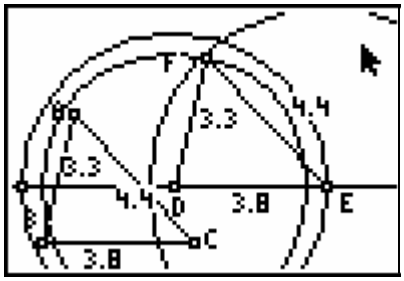

Figure 4

- Press F5 and select **Measure- D. & length**. Measure the corresponding sides of ΔABC and ΔDEF to validate the corresponding sides have the same measure and are congruent (figure 4).
- Press F5 and select **Hide/show- objects**. Hide the circles used in construction and the measures of the side lengths (figure 5).
- Measure corresponding angles. Two angles are sufficient because the third angles will be congruent if two angles of one triangle are congruent to two angles of a second triangle as shown in figure 5.
- It has been shown that corresponding sides of ΔABC and ΔDEF are congruent and it followed that corresponding angles are also congruent. By definition of congruent polygons, ΔABC ≅ ΔDEF.
- It was sufficient to show three sides of one triangle congruent to three corresponding sides of a second triangle for the two triangles to be congruent (SSS).

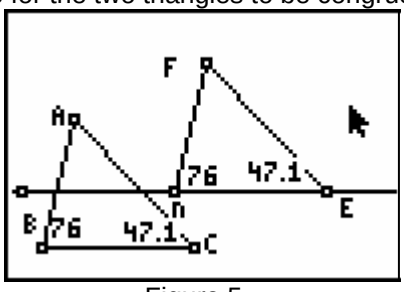

Figure 5

2. Construction of ΔDEF congruent to ΔABC given two sides and an included angle (SAS) using the TI-83+ graphing calculator with Cabri Junior™ software:

- Press F2 and select **Triangle** followed by ENTER. Draw a scalene triangle and label its vertices with the letters A, B, and C (figure 1).
- Press F2 and select **Line** followed by ENTER. Draw a line on the screen, locate point D on the line, and label the point with the letter D as shown in figure 1.
- Press F2, select **Circle,** and draw circle B with a radius less than the side length of either segment AB or BC so that the circle intersects both sides of angle ABC. Mark and label the points R and T (figure 2).
- Press F3, select **Compass,** and construct circle B with radius BT and press ENTER. Then move the circle so that its center coincides with point D on the given line and press ENTER to construct circle D. Label the point of intersection of circle D with the line, point Y (figure 2).

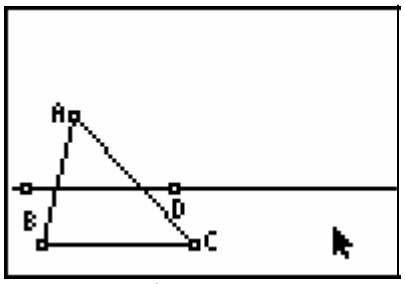

Figure 1

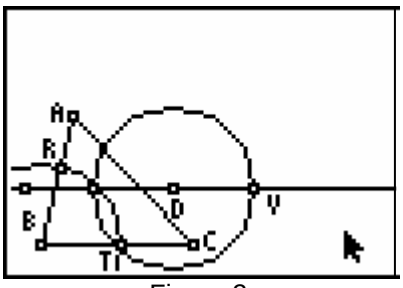

Figure 2

- Press F3 and select **Compass** followed by ENTER. Select radius RT and press ENTER. Move the center of the circle to coincide with point Y and press ENTER.
- Mark and label point W, the intersection of circle D and circle Y (figure 3).
- Press F2, select **Line**, and draw line DW as shown in figure 3.

- ∠WDY has been constructed congruent to ∠ABC and can be verified as follows:

| | |
|---|---|
| RB = WD | Circle B ≅ circle D and congruent circles have equal radii. |
| BT = DY | Circle B ≅ circle D and congruent circles have equal radii. |
| RT = WY | Circle T ≅ circle Y and congruent circles have equal radii. |
| ΔRBT ≅ ΔWDY | SSS |
| ∠RBT ≅ ∠WDY | CPCTC (Corresponding parts of congruent triangles are congruent.) |

- Press F5 and select **Hide/Show- objects** followed by ENTER. Move the cursor to each object you want to hide (construction circles) and press ENTER after each selection as shown in figure 4 below.

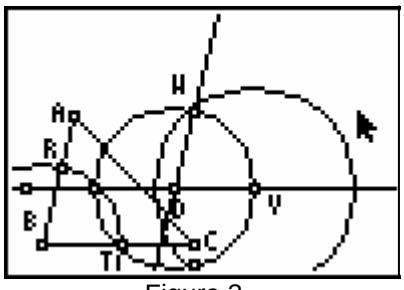

Figure 3

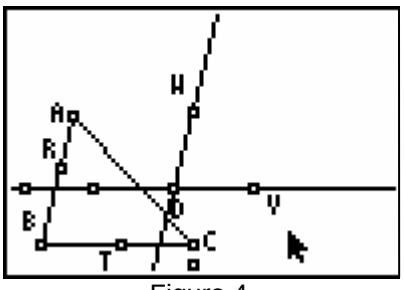

Figure 4

- Press F3, select **Compass**, and construct circle B with radius BC. Move the cursor to point D and press ENTER to construct circle D with radius BC.

- Press F2, select **Point-intersection**, and mark/label point E where circle D intersects the line (figure 5).
- Press F5, select **Hide/Show-objects**, and hide circle D as shown in figure 6.
- Press F3, select **Compass**, and construct circle B with radius AB. Move the cursor to point D and press ENTER to construct circle D with radius AB (figure 6).
- Press F2, select **Point-intersection**, and mark the point of intersection of circle D with line DW. Label this point of intersection, point F.

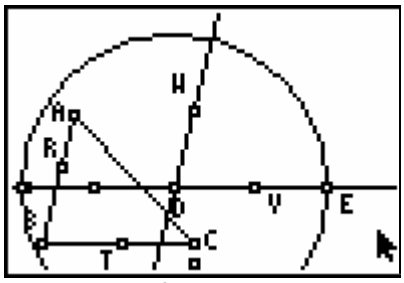

Figure 5

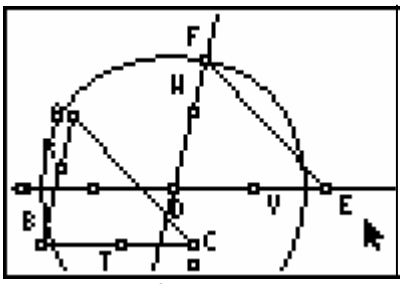

Figure 6

- Press F5, select **Hide/Show-objects,** and hide the construction circle D as shown in figure 7.

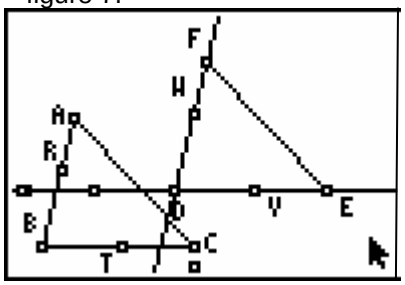

Figure 7

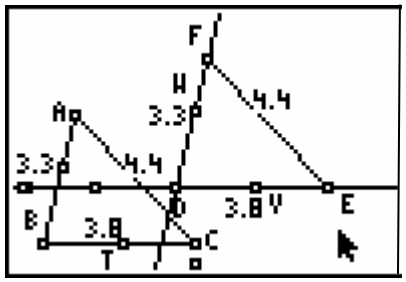

Figure 8

- Press F5, select **Measure-D. & length**, and measure the corresponding sides of ΔABC and ΔFDE. ΔABC ≅ ΔFDE by SSS as shown in figure 8. ΔFDE was constructed congruent to ΔABC by constructing segment DF ≅ segment AB, ∠FDE ≅ ∠ABC, and segment DE ≅ segment BC. Therefore, two sides and the included angle of one triangle congruent to two sides and the included angle of a second triangle would be sufficient for the triangles to be congruent.

3. Construction of ΔDEF congruent to ΔABC given two angles and an included side (ASA) using the TI-83+ graphing calculator with Cabri Junior™ software:
   - Press F2 and select **Triangle** followed by ENTER. Draw a scalene triangle and label its vertices with the letters A, B, and C (figure 1).
   - Press F2 and select **Line** followed by ENTER. Draw a line on the screen, locate point D on the line, and label the point with the letter D as shown in figure 1.
   - Press F2, select **Circle,** and draw circle B with a radius less than the side length of either segment AB or BC so that the circle intersects both sides of angle ABC. Mark and label the points R and T (figure 2).
   - Press F3, select **Compass,** and construct circle B with radius BT and press ENTER. Then move the circle so that its center coincides with point D on the given line and press ENTER. Label the point of intersection of circle D with the line, point Y (figure 2).

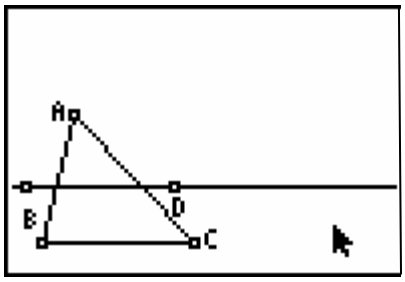

Figure 1

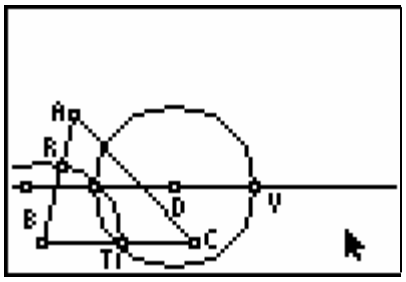

Figure 2

- Press F3 and select **Compass** followed by ENTER. Select radius RT and press ENTER. Move the center of the circle to coincide with point Y and press ENTER.
- Mark and label point W, the intersection of circle D and circle Y (figure 3).
- Press F2, select **Line**, and draw line DW as shown in figure 3.
- ∠WDY has been constructed congruent to ∠ABC.

| | |
|---|---|
| RB = WD | Circle B ≅ circle D; congruent circles have equal radii. |
| BT = DY | Circle B ≅ circle D; congruent circles have equal radii. |
| RT = WY | Circle T ≅ circle Y; congruent circles have equal radii. |
| ΔRBT ≅ ΔWDY | SSS |
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- Press F5 and select **Hide/Show- objects** followed by ENTER. Move the cursor to each object you want to hide (construction circles) and press ENTER after each selection as shown in figure 4 below.

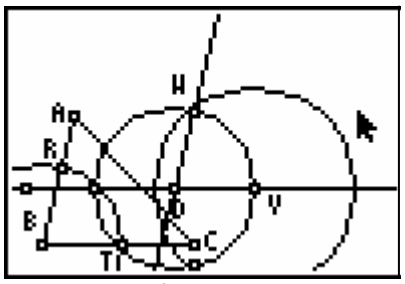

Figure 3

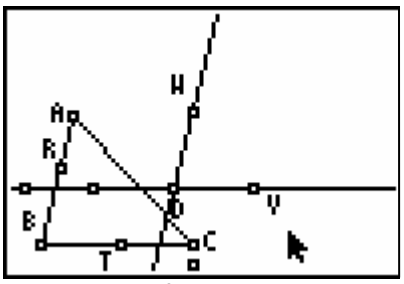

Figure 4

- Press F3, select **Compass**, and construct circle B with radius BC. Press ENTER, move the center to coincide with point D, and press ENTER again. Mark/label the point of intersection of circle D with the given line, point E. Now BC = DE because radii of congruent circles are congruent.
- Press F5, select **Hide/show-object**, and hide circle D as shown in figure 6.

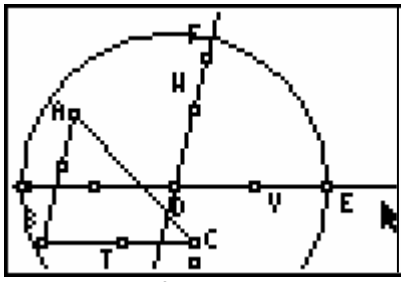

Figure 5

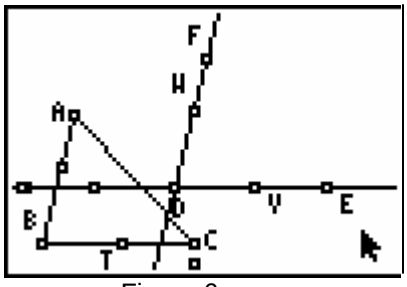

Figure 6

- Construct ∠DEF ≅ ∠ACB as shown in figure 7.

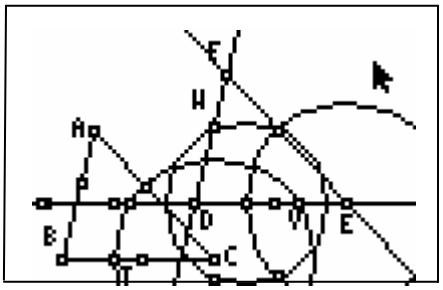

Figure 7

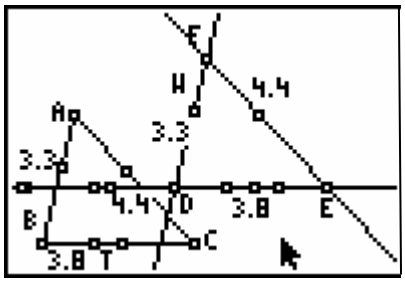

Figure 8

- ΔDEF can be verified to be congruent to ΔABC by SSS as shown in figure 8. These triangles were constructed using ∠ FDE ≅ ∠ABC, segment DE ≅ segment BC, and ∠ FED ≅ ∠ACB. Therefore, two triangles can be shown congruent using two angles and the included side of one triangle congruent to two angles and the included side of a second triangle.

**Extension:**
Use a TI-83+ graphing calculator with Cabri Junior™ software to explore the following:
Can two triangles be shown congruent using AAS? Make a conjecture and verify your conjecture using constructions on your graphing calculator.

**References:**
www.education.ti.com (TI-83 Plus Cabri® Jr. App Guide Book)

T$^3$• Teachers Teaching with Technology™ (2003).*Cabri Junior™ Participant Packet.* Dallas, TX: Texas Instruments Incorporated.