Hole-in-One!

Purpose:
Participants will use line reflections to determine how to make a “hole-in-one” on a miniature golf course. They will justify their solution using definitions, axioms, postulates, and/or theorems.

Procedure.
Participants will use a diagram of a miniature golf course with given positions of the ball and cup. They will use symmetry, reflections about a line, a law of physics involving the angle of incidence and angle of reflection, and problem-solving strategies to solve this real-world problem. Solution(s) will be justified using definitions, axioms, postulates, and/or theorems of geometry.

TExES Mathematics 4-8 Competencies. The beginning teacher:

III.009.A Understands concepts and properties of points, lines, planes, angles, lengths, and distances.
III.009.B Analyzes and applies the properties of parallel and perpendicular lines.
III.009.C Uses the properties of congruent triangles to explore geometric relationships and prove theorems.
III.010.C Uses a variety of representations (e.g., numeric, verbal, graphic, symbolic) to analyze and solve problems involving two-and three-dimensional figures such as circles, triangles, polygons, cylinders, prisms, and spheres.
III.011.A Describes and justifies geometric constructions made using a reflection device and other appropriate technologies.
III.011.B Uses translations, reflections, glide-reflections, and rotations to demonstrate congruence and to explore the symmetries of figures.

TEKS Mathematics Objectives. The student is expected to:

4.8A Identify right, acute, and obtuse angles.
4.8B Identify models of parallel and perpendicular lines.
4.9A Demonstrate translations, reflections, and rotations using concrete models.
4.9B Use translations, reflections, and rotations to verify that two shapes are congruent.
4.9C Use reflections to verify that a shape has symmetry.
5.8A Sketch the results of translations, rotations, and reflections.
6.6A Use angle measurements to classify angles as acute, obtuse, or right.
6.6B Identify relationships involving angles in triangles and quadrilaterals.
6.8C Measure angles.
7.6B Use properties to classify shapes including triangles, quadrilaterals, pentagons, and circles.
8.6 Use transformational geometry to develop spatial sense.

Terms.
Reflection, line/reflectional/bilateral symmetry, line of symmetry, congruent, similar, angle of incidence, angle of reflection, vertical angles, perpendicular, perpendicular bisector, angle bisector

Materials.
- Straight edge
- Protractor
- Compass
- Mira (optional)

Transparencies.
- *Hole in One!*

Activity Sheet(s).

- *Hole in One!*

Procedure.

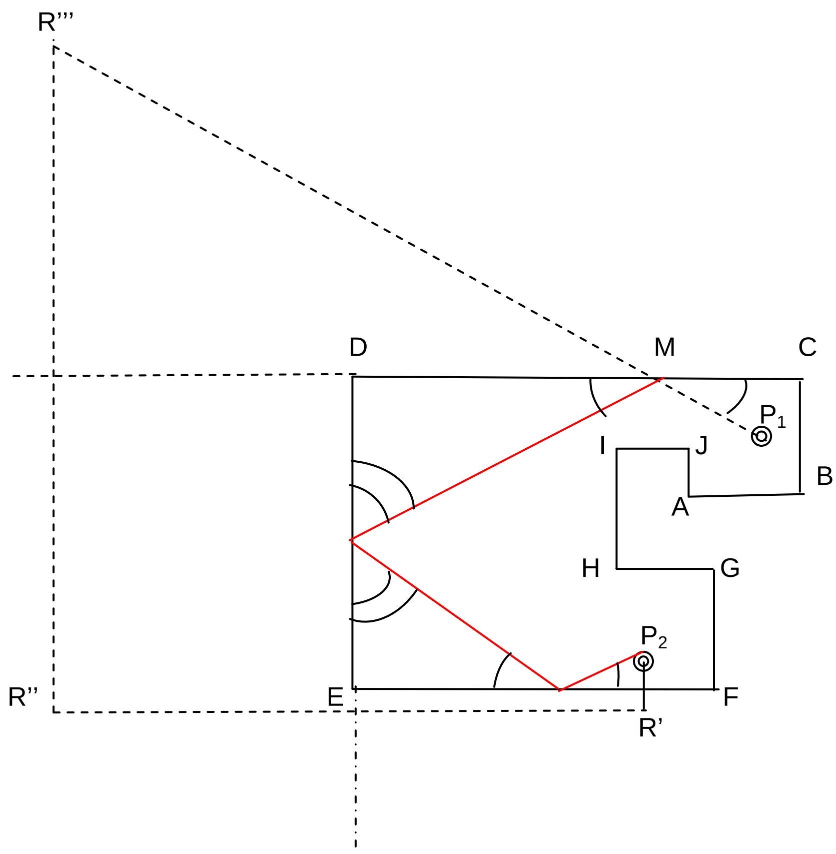
Steps	Questions/Math Notes
1. Have participants read the problem on the Transparency *Hole in One!* . Ask some to share strategies for making a "hole-in-one". Remind them to draw upon the axioms, definitions, and/or theorems from geometry.	Ask questions to foster thinking about the problem and the mathematics involved. *What does it mean to make a "hole-in-one"?* *How will the law of physics "the angle of incidence is congruent to the angle of reflection" help you in playing the game?* *How could you determine the angle of incidence?*
2. Ask participants to work in pairs on the problem. Have materials on the table for them to select from. Select a few groups to show how they solved the problem. Ask others in the larger group to justify statements made by the presenters using the language, axioms, and/or theorems of geometry.	*What problem-solving strategy or strategies did you use?* *How did transformations help you solve this problem? Explain.* *What are some different strategies for reflecting a point about a line?* *How can you validate that a point has been reflected about a line?* *What is this line called? What is the reflection of the point called?* *Is it possible to make a "hole-in-one" on this course in fewer than 3 bounces? If so, how?*

Solution.

1. The problem-solving strategy "working backwards" can be used to solve this problem. Begin by reflecting the cup at point P_2 about the wall or segment EF. Label the image of P_2 , R'. Reflect R' about the extension of segment DE and label its image, R''. Extend segment DC and reflect R'' about this ray labeling its image, R''' as shown in the diagram. Connect R''' to the ball's position at point P_1. To achieve a "hole-in-one", you should hit the ball so that it strikes the wall at point M and continues to bounce off the walls at points indicated on the walls where the red segments intersect.

2. Answers will vary.

3. Since the angle of incidence has the same measure as the angle of reflection, the angles at point M noted by one arc between the sides have equal measures. When the ball strikes the wall represented by segment DE in the diagram, the angles marked with two arcs (angle of incidence and angle of reflection) have the same measure. Since angle D and angle E are right angles, right triangles are formed with the hypotenuse of each opposite a right angle. If two angles of one triangle are congruent to two angles of a second triangle, the third angles are congruent and have the same measure. Therefore, the angle of incidence at point M has the same measure as the angle of incidence on segment EF. This angle of incidence will have the same measure as the angle of reflection that makes a "hole in one". The angle is about 30°.



Reference:

Serra, Michael(1997). *Discovering Geometry, An Inductive Approach.* Berkeley, CA: Key Curriculum Press.