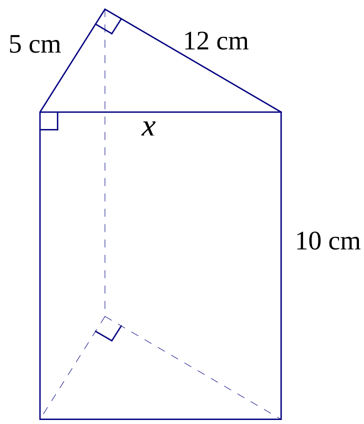
SURFACE AREA AND VOLUME

Surface Area and Volume of a Prism

Exercises (page-9):

1.



a) Let the unknown side of the base (the hypotenuse) be x cm. by the Pythagorean theorem, $5^2 + 12^2 = x^2$.
That is, $x^2 = 169$ and $x = 13$.
Hence, the missing side is 13 cm.

b) The lateral area is $L = Ph = (5+12+13)10 = 300 \text{ cm}^2$.

c) Firstly find the area of the base: $B = \frac{1}{2}(5)(12) = 30 \text{ cm}^2$.

The total surface area is: $T = L + 2B = 300 + 2(30) = 360 \text{ cm}^2$.

d) The volume is: $V = Bh = (30)(10) = 300 \text{ cm}^3$.

2. a) The lateral area is: $L = Ph = (4+7+4+7)15 = (22)15 = 330 \text{ cm}^2$.

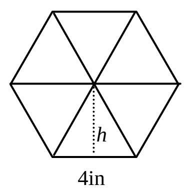
b) The area of the base is: $B = (4)(7) = 28 \text{ cm}^2$.

Hence, the total area is: $T = L + 2B = 330 + 2(28) = 386 \text{ cm}^2$.

c) The volume is: $V = Bh = (28)(15) = 420 \text{ cm}^3$.

3. a) The lateral area is: $L = Ph = (4 \cdot 6) \cdot 9 = 24 \cdot 9 = 216 \text{ in}^2$.

b) The base is formed of 6 congruent equilateral triangles. To find the base area, we need the area of one of the triangles. Let the height of one of them be *h*. Since triangle formed by drawing the height is a 30^0-60^0-90^0 triangle, we get $h = 2\sqrt{3}$ in. Then, the area of a triangle is $A = \frac{1}{2}(4)\left(2\sqrt{3}\right) = 4\sqrt{3}$ in^2.

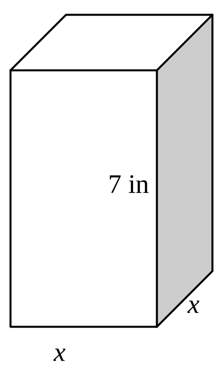


The area of the base is: $B = (6)\left(4\sqrt{3}\right) = 24\sqrt{3}$ in^2.

Hence, the total area is: $T = L + 2B = 216 + 2\left(24\sqrt{3}\right) = \left(216 + 48\sqrt{3}\right)$ in^2.

c) The volume is: $V = Bh = \left(24\sqrt{3}\right)(9) = 216\sqrt{3}$ cm^3.

4. Let one side of the base be *x* in.

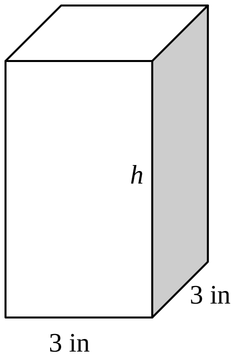


a) Since the volume is $V = Bh = \left(x^2\right)7 = 175$, we have: $x^2 = \dfrac{175}{7} = 25 \Rightarrow x = 5$ (Notice that *x* can not take negative values.).
Hence, one side of the base is 5 in.

b) The lateral area is: $L = Ph = (4 \cdot 5) \cdot 7 = 140$ in^2.

c) The total surface area is: $T = L + 2B = 140 + 2\left(5^2\right) = 140 + 50 = 190$ in^2.

5.

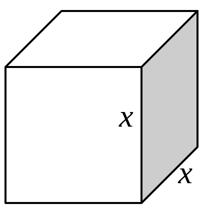


a) The volume is: $V = Bh = \left(3^2\right)h = 9h = 108$. Hence, $h = \dfrac{108}{9} = 12 \text{ in}$.

b) The lateral area is: $L = Ph = (4 \cdot 3) \cdot 12 = 144 \text{ in}^2$.

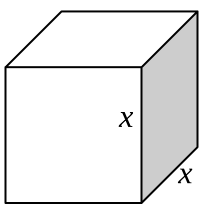
c) The total surface area is: $T = L + 2B = 144 + 2\left(3^2\right) = 144 + 18 = 162 \text{ in}^2$.

6.



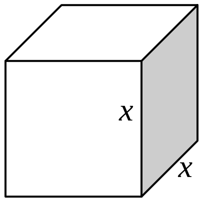
Let one side of the cube be x cm. The lateral area of this cube is: $L = Ph = (4x)x = 4x^2$.
That is, $4x^2 = 144 \;\Rightarrow\; x^2 = 36 \;\Rightarrow\; x = 6$.
The length of the edge is 6 cm.

7.



The total surface area of the cube is: $T = L + 2B = \left(4x^2\right) + 2\left(x^2\right) = 6x^2$. Since
$6x^2 = 96 \;\Rightarrow\; x^2 = 16 \;\Rightarrow\; x = 4$, the length of an edge is 4 m.

8.



The volume of this cube is: $V = Bh = \left(x^2\right)x = x^3 = 343$. Thus, $x = 7$. The length of an edge is 7 cm.

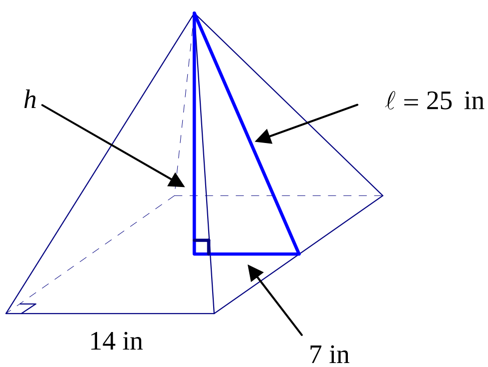
Surface Area and Volume of a Pyramid

Exercises (page-17):

1. a) The lateral area is $L = \frac{1}{2}lP = \frac{1}{2}(25)(4 \cdot 14) = 700 \text{ in}^2$.

b) The total surface area is: $T = L + B = 700 + \left(14^2\right) = 896 \text{ in}^2$.

c) We need to find the height of the pyramid. The following figure illustrates the situation.



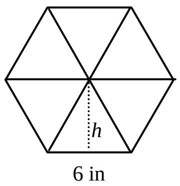
Let's use the Pythagorean Theorem in the right triangle formed by drawing the height;

$$h^2 + 7^2 = 25^2 \Rightarrow h^2 = 576 \Rightarrow h = 24 \text{ in}.$$

Hence, the volume is: $V = \frac{1}{3}Bh = \frac{1}{3}\left(14^2\right)24 = 1568 \text{ in}^3$.

2. a) The lateral area of the pyramid is: $L = \frac{1}{2}lP = \frac{1}{2}(20)(6 \cdot 6) = 360 \text{ in}^2$.

b) We need to find the base area first.

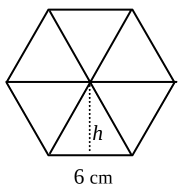


In the figure above, $h = 3\sqrt{3}$ in and the area of one triangle is $A = \frac{1}{2}(6)\left(3\sqrt{3}\right) = 9\sqrt{3}\ \text{in}^2$.

Hence, the base area is $B = 6\left(9\sqrt{3}\right) = 54\sqrt{3}\ \text{in}^2$.

The total surface area is: $T = L + B = 360 + \left(54\sqrt{3}\right) = \left(360 + 54\sqrt{3}\right)\ \text{in}^2$.

3. We need the base area of the given pyramid.



The height of a triangle is $h = 3\sqrt{3}$ cm and the area of a triangle is
$A = \frac{1}{2}(6)\left(3\sqrt{3}\right) = 9\sqrt{3}\ \text{cm}^2$.

Hence, the base area is $B = 6\left(9\sqrt{3}\right) = 54\sqrt{3}\ \text{cm}^2$.

The volume of the pyramid is: $V = \frac{1}{3}Bh = \frac{1}{3}\left(54\sqrt{3}\right)10 = 180\sqrt{3}\ \text{cm}^2$.

4. Since the base is a square, if we let one side of the base be *x*, then the base area is: $B = x^2$.

The volume of the pyramid is: $V = \frac{1}{3}Bh = \frac{1}{3}\left(x^2\right)10 = 120$. So, $x^2 = 36$ and $x = 6$.

Hence, one side of the base is 6 cm.

5. The lateral area is: $L = \frac{1}{2}lP = \frac{1}{2}(8)P = 80$. Hence, $P = 20$. Since the base is a square, if one side of the square is *x* in, then the perimeter is $4x$ in.

We have; $P = 4x = 20 \ \Rightarrow\ x = 5$. One side of the base is 5 in.

Surface Area and Volume of a Cylinder

Exercises (page-21):

1. $r = 10$ and $h = 4$ in ;
a) The lateral area is: $L = Ch = (2\pi \cdot 10) \cdot 4 = 80\pi \text{ in}^2$.

b) The total surface area is: $T = L + 2B = 80\pi + 2\left(\pi \cdot 10^2\right) = 280\pi \text{ in}^2$.

c) The volume is: $V = Bh = (100\pi) \cdot 4 = 400\pi \text{ in}^3$.

2. $d = 14$, $r = 7$ and $h = 9$ in ;

a) The lateral area is: $L = Ch = (\pi \cdot 14) \cdot 9 = 126\pi \text{ in}^2$.

b) The total surface area is: $T = L + 2B = 126\pi + 2\left(\pi \cdot 7^2\right) = 224\pi \text{ in}^2$.

c) The volume is: $V = Bh = (49\pi) \cdot 9 = 441\pi \text{ in}^3$.

3. $d = 5$, $r = \frac{5}{2}$ and $h = 6$ m ;

a) The lateral area is: $L = Ch = (\pi \cdot 5) \cdot 6 = 30\pi \text{ m}^2$.

b) The total surface area is: $T = L + 2B = 30\pi + 2\left(\pi \cdot \left(\frac{5}{2}\right)^2\right) = \frac{85}{2}\pi \text{ m}^2$.

c) The volume is: $V = Bh = \left(\frac{25}{4}\pi\right) \cdot 6 = \frac{75}{2}\pi \text{ m}^3$.

4. $h = 5$ in and $\text{V} = 245\pi \text{ in}^3$;

a) $V = Bh = \left(\pi r^2\right) \cdot 5 = 245\pi \Rightarrow r^2 = \frac{245}{5} = 49 \Rightarrow r = 7$.
Hence, the radius is 7 in.

b) The lateral area is: $L = Ch = (2\pi \cdot 7) \cdot 5 = 70\pi \text{ in}^2$.

c) The total surface area is: $T = L + 2B = 70\pi + 2\left(\pi \cdot 7^2\right) = 168\pi \text{ in}^2$.

5. $d = 16$, $r = 8$ cm, and $L = 192\pi \text{ cm}^2$;

a) $L = Ch = (\pi \cdot 16) \cdot h = 192\pi \;\Rightarrow\; h = \dfrac{192}{16} = 12$.
Hence, the height of the pyramid is 12 cm.

b) The total surface area is: $T = L + 2B = 192\pi + 2\left(\pi \cdot 8^2\right) = 320\pi \text{ cm}^2$.

c) The volume is: $V = Bh = (64\pi) \cdot 12 = 768\pi \text{ cm}^3$.

Surface Area and volume of a Cone

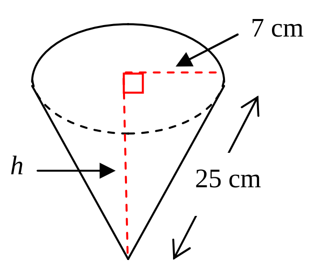
Exercises (page-26):

1. $r = 7$ and $l = 25 \text{ cm}$;

a) The lateral area is: $L = \tfrac{1}{2}\ell C = \tfrac{1}{2}(25)(2\pi \cdot 7) = 175\pi \text{ cm}^2$.

b) The total surfaces area is: $T = L + B = 175\pi + 49\pi = 224\pi \text{ cm}^2$.

c)

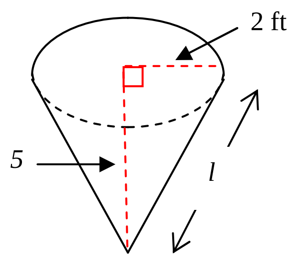


If we apply the Pythagorean Theorem in the right triangle as we did before, then we get $h = 24 \text{ cm}$.

The volume is: $V = \tfrac{1}{3}Bh = \tfrac{1}{3}(49\pi)24 = 392\pi \text{ cm}^3$.

2. $d = 4,\ r = 2,$ and $h = 5$ ft ;

a) We need to find the slant height.



By Pythagorean Theorem, $l^2 = 25 + 4 = 29$; that is, $l = \sqrt{29}$ ft .
The lateral area is: $L = \frac{1}{2}\ell C = \frac{1}{2}\left(\sqrt{29}\right)\left(2\pi \cdot 2\right) = 2\sqrt{29}\pi$ ft^2.

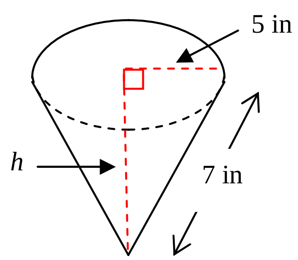
b) The total surfaces area is: $T = L + B = 2\sqrt{29}\pi + 4\pi = \left(2\sqrt{29} + 4\right)\pi$ ft^2.

c) The volume is: $V = \frac{1}{3}Bh = \frac{1}{3}\left(4\pi\right)5 = \frac{20}{3}\pi$ ft^3.

3. $r = 5$ in and $L = 35\pi$ in^2;

a) $L = \frac{1}{2}\ell C = \frac{1}{2}l\left(2\pi \cdot 5\right) = 35\pi \ \Rightarrow\ l = \frac{35}{5} = 7$.
The slant height is 7 in.

b)



By the Pythagorean Theorem, $h^2 + 25 = 49 \ \Rightarrow\ h^2 = 24 \ \Rightarrow\ h = 2\sqrt{3}$.
The height is $2\sqrt{3}$ in.

c) The total surface area is: $T = L + B = 35\pi + \left(\pi \cdot 5^2\right) = 60\pi$ in^2.

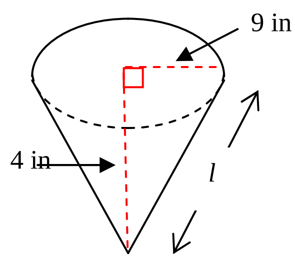
d) The volume is: $V = \frac{1}{3}Bh = \frac{1}{3}\left(25\pi\right)2\sqrt{3} = \frac{50\sqrt{3}}{3}\pi$ in^3.

4. $d = 18,\ r = 9 \text{ in},$ and $V = 108\pi \text{ in}^3$;

a) $V = \frac{1}{3}Bh = \frac{1}{3}(81\pi)h = 108\pi \Rightarrow h = \frac{108}{27} = 4 \text{ in}$.

The height is 4 in.

b)



By the Pythagorean Theorem, $16 + 81 = l^2 \Rightarrow l^2 = 97 \Rightarrow l = \sqrt{97}$.

The slant height is $\sqrt{97}$ in.

c) The lateral area is: $L = \frac{1}{2}\ell C = \frac{1}{2}\left(\sqrt{97}\right)(2\pi \cdot 9) = 9\sqrt{97}\pi \text{ in}^2$.

d) The total surface area is: $T = L + B = 9\sqrt{97}\pi + \left(\pi \cdot 9^2\right) = \left(81 + 9\sqrt{97}\right)\pi \text{ in}^2$.