PERIMETER AND AREA

Circumference
Exercises:

1.

OBJECT	DIAMETER	CIRCUMFERENCE	$\frac{\text{CIRCUMFERENCE}}{\text{DIAMETER}}$
Lid (outer rim)	6.1 cm	19.3 cm	3.164
Soda Can (top rim)	5.5 cm	17.2 cm	3.127
Drinking Glass (top rim)	8.2 cm	25.7 cm	3.134
Jar (top rim)	4.7 cm	14.9 cm	3.170
Mixing Bowl (top rim)	21.6 cm	67.8 cm	3.139
Bucket (top rim)	31.0 cm	97.5 cm	3.145

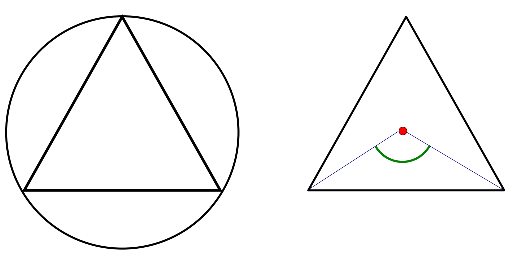
2.

OBJECT	DIAMETER	CIRCUMFERENCE	$\frac{\text{CIRCUMFERENCE}}{\text{DIAMETER}}$
Vase (top rim)	7.5 cm	23.9 cm	3.187
Coffee Cup (top rim)	4.2 cm	13.3 cm	3.167
Compact Disc (top rim)	11.9 cm	37.4 cm	3.143

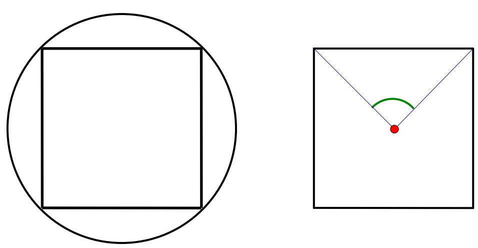
3. It is easily seen that the numbers in the last column are very close to the number $\pi \approx 3.14159265358979323846264338327950288419716939937510...$. If the objects were exactly circular and the measurement were accurate, then the ratio would be same as the number π.

Area of a Regular Polygon

Exercises:
1.
a)



b)



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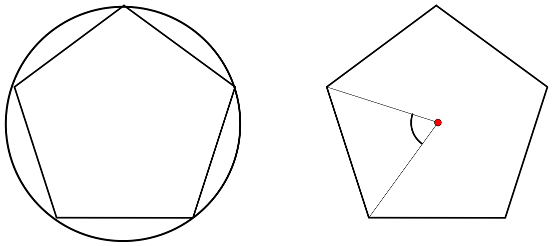

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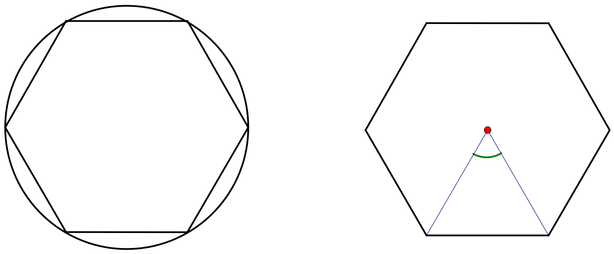
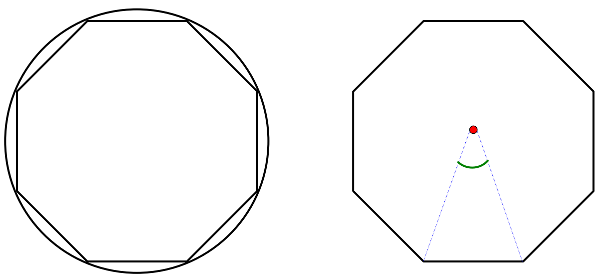


e)



2. a) Regular Decagon: $\text{central angle} = \frac{360^0}{10} = 36^0$.

b) Regular Dodecagon: $\text{central angle} = \frac{360^0}{12} = 30^0$.

c) Regular 36-gon: $\text{central angle} = \frac{360^0}{36} = 10^0$.

d) Regular 120-gon: $\text{central angle} = \frac{360^0}{120} = 3^0$.

EXERCISES (at the end of the section):

1. $\text{Perimeter of a square} = 4(\text{side length}) = 4(100) = 400 \text{ cm}$.

2. Perimeter of a square = 4(side length) $\Rightarrow$ side length $= \dfrac{\text{perimeter}}{4} = \dfrac{100}{4} = 25$ cm .

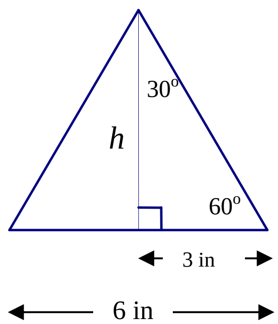
3. Area of a square $= (\text{side length})^2 = (100)^2 = 10000 \text{ cm}^2$.

4. Area of a square $= (\text{side length})^2 \Rightarrow$ side length $= \sqrt{\text{Area}} = \sqrt{100} = 10$ cm .

5. Perimeter of an equilateral triangle = 3(side length) = 3(6) = 18 in .

6. Perimeter of an equilateral triangle = 3(side length) $\Rightarrow$ side length $= \dfrac{\text{perimeter}}{3} = \dfrac{6}{3} = 2$ in .

7. Since one side of the equilateral triangle is 6 inches, the height is: $h = 3\sqrt{3}$.



Area of a triangle $= \frac{1}{2}(\text{side length})(\text{height}) = \dfrac{1}{2}(6)\left(3\sqrt{3}\right) = 9\sqrt{3} \text{ cm}^2$.

8. Perimeter of the given rectangle is: $P = 2l + 2w = 2(7) + 2(5) = 24$ m .

9. Area of the given rectangle is : $A = (\text{length})(\text{width}) = (7)(5) = 35 \text{ m}^2$.

10. $P = 2l + 2w = 2(8) + 2w = 40 \Rightarrow 16 + 2w = 40 \Rightarrow 2w = 24 \Rightarrow w = 12$ in .

11. $A = (\text{length})(\text{width}) = (8)(w) = 40 \Rightarrow w = 5$ in .

12. Perimeter of a regular hexagon is: $P = 6(\text{side length}) = 6(6) = 36$ cm .

13. The apothem of the hexagon is $a = 6\sqrt{3}$ and the perimeter is $P = 36$ cm . Thus,

$$A = \tfrac{1}{2} aP = \tfrac{1}{2}\left(3\sqrt{3}\right)(36) = 54\sqrt{3} \text{ cm}^2 .$$

14. $P = 6(\text{side length}) = 6 \Rightarrow$ side length $= \dfrac{6}{6} = 1$ cm .

15. $P = 2l + 2w = 2l + 2(12) = 54 \Rightarrow 2l + 24 = 54 \Rightarrow 2l = 30 \Rightarrow l = 15$ in .

16. $A = (\text{length})(\text{width}) = (12)(w) = 54 \Rightarrow w = \frac{9}{2}$ in .

17. Area of the square is: $A = (\text{side length})^2 = (x+3)^2 = \left(x^2 + 6x + 9\right)$ ft^2 .

18. Area of the given rectangle is:
$A = (\text{length})(\text{width}) = (x+7)(x+3) = \left(x^2 + 10x + 21\right)$ft^2 .

19. a) $A = (\text{length})(\text{width}) = (x+7)(x+3) = x^2 + 10x + 21 = 165 \Rightarrow x^2 + 10x - 144 = 0$

$\Rightarrow (x+18)(x-8) = 0$

$\Rightarrow x = -18$ or $x = 8.$

However, $x = -18 \Rightarrow x + 3 = -15$ and $x + 7 = -11$. Since the measure of the length can not be negative, we conclude that $x = 8$.

b) $width = x + 3 = 8 + 3 = 11$ ft .

c) $length = x + 7 = 8 + 7 = 15$ ft .

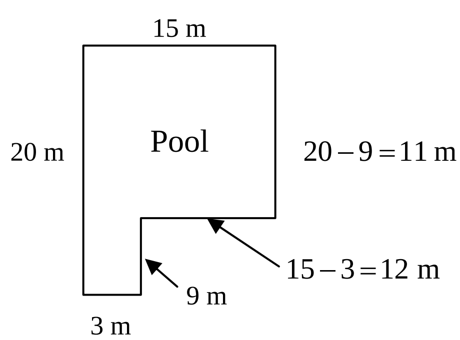
20. a) $P = 2l + 2w = 2(x+7) + 2(x+3) = 104 \Rightarrow 4x + 20 = 104 \Rightarrow 4x = 84 \Rightarrow x = 21$ ft .

b) $width = x + 3 = 21 + 3 = 24$ ft .

c) $length = x + 7 = 21 + 7 = 28$ ft .

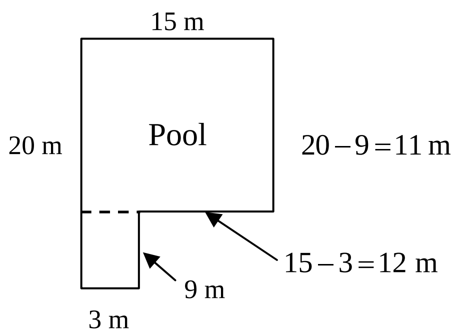
21. $A = (l)(w) = (l)(x+1) = 2x^2 + 5x + 3 \Rightarrow \text{length} = \frac{2x^2 + 5x + 3}{x+1} = \left(x + 4 - \frac{1}{x+1}\right)$ ft .

22.



Hence, the perimeter of the pool is: $P = 20 + 15 + 11 + 9 + 3 = 58$ m .

23. In the figure below, there are 2 rectangles. The bigger rectangle has dimensions 15 and 11 meters, and the small rectangle has dimensions 9 and 3 meters.



The area of the pool is the sum of the areas of the 2 rectangles:
$A = \mathit{Area}(\text{small rectangle}) + \mathit{Area}(\text{big rectangle}) = (15 \cdot 11) + (9 \cdot 3) = 165 + 27 = 192 \text{ m}^2$.

24. Circumference of the circle is: $C = 2\pi r = 2\pi(16) = 32\pi$ m.

25. The diameter is 16m implies that the radius is 8m. Hence, $C = 2\pi r = 2\pi(8) = 16\pi$ m.

26. Area of the circle is: $A = \pi r^2 = \pi(16)^2 = 256\pi$ m^2.

27. $\text{Diameter} = 16m \Rightarrow \text{radius} = 8 \text{ m}$. Thus, $A = \pi r^2 = \pi(8)^2 = 64\pi$ m^2.

28. $A = \pi r^2 = 16\pi \Rightarrow r^2 = 16 \Rightarrow r = \pm 4$.
Since the radius can not be a negative value, we conclude that $r = 4$ m.

29. $C = 2\pi r = 16\pi \Rightarrow r = \dfrac{16\pi}{2\pi} = 8 \text{ m}$.

30. $C = 2\pi r = 36\pi \Rightarrow r = \dfrac{36\pi}{2\pi} = 18 \text{ cm}$ and $A = \pi r^2 = \pi(18)^2 = 324\pi \text{ cm}^2$.

31. $A = \pi r^2 = 36\pi \Rightarrow r^2 = 36 \Rightarrow r = 6 \text{ cm}$ and $C = 2\pi r = 2\pi(6) = 12\pi \text{ cm}$.

32. Area of the given parallelogram is: $A = (\text{base})(\text{height}) = (5)(6) = 30 \text{ cm}^2$.

33. Area of the given triangle is: $A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(5)(6) = 15 \text{ cm}^2$.

34. Area of the given trapezoid is: $A = \frac{1}{2}\left(b_1 + b_2\right)h = \frac{1}{2}(5 + 12)(6) = 51 \text{ cm}^2$.

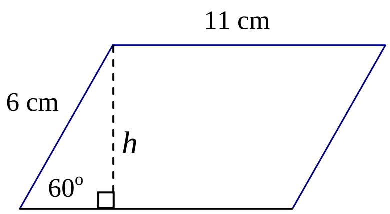
35. $A = (\text{base})(\text{height}) = (8)(\text{height}) = 28 \;\Rightarrow\; \text{height} = \dfrac{28}{8} = \dfrac{7}{2}\text{ in}^2$.

36. $A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(8)(\text{height}) = 28 \;\Rightarrow\; \text{height} = \dfrac{28}{4} = 7\text{ in}^2$.

37. $A = \frac{1}{2}\left(b_1 + b_2\right)h = \frac{1}{2}(8+12)h = 28 \;\Rightarrow\; 10h = 28 \;\Rightarrow\; h = \dfrac{14}{5}\text{ in}$.

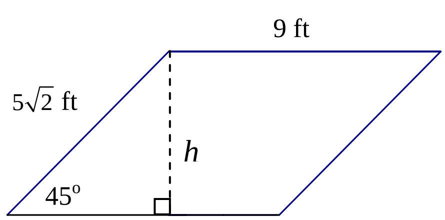
38. a) Area of the given triangle is: $A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(7)(12) = 48\text{ cm}^2$.

b) In the figure below, $h = 6\left(\dfrac{\sqrt{3}}{2}\right) = 3\sqrt{3}\text{ cm}$ because of the relations in 30^0-60^0-90^0 triangle (*h* is the longer leg of the triangle).



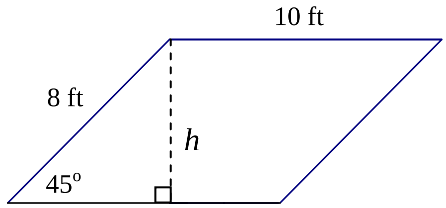
Hence, the area of the parallelogram is: $A = (\text{base})(\text{height}) = (11)(3\sqrt{3}) = 33\sqrt{3}\text{ cm}^2$.

c) If we draw the height of the given parallelogram as in the figure below, we get an isosceles right triangle with hypotenuse $5\sqrt{2}$ ft. Hence, one side of that right triangle is: $h = \dfrac{5\sqrt{2}}{\sqrt{2}} = 5$ ft.



Thus, the area is: $A = (\text{base})(\text{height}) = (9)(5) = 45\text{ ft}^2$.

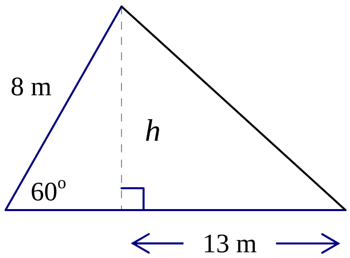
d) In the figure below, *h* is one side of an isosceles right triangle whose hypotenuse is 8 ft. That is, $h = \frac{8}{\sqrt{2}} = 4\sqrt{2}$ ft.



Hence, the area is: $A = \text{(base)(height)} = (10)\left(4\sqrt{2}\right) = 40\sqrt{2}\ \text{ft}^2$.

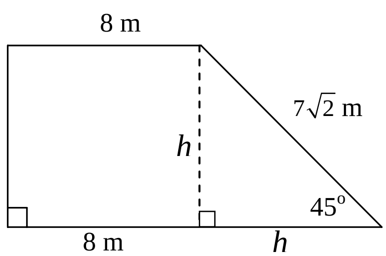
e) The area of the given trapezoid is: $A = \frac{1}{2}\left(b_1 + b_2\right)h = \frac{1}{2}(20+7)4 = 54\ \text{cm}^2$.

f) In the figure below, since *h* is the longer leg of a 30^0-60^0-90^0 triangle with a hypotenuse of $8\ \text{m}$, $h = 8\left(\frac{\sqrt{3}}{2}\right) = 4\sqrt{3}$ m.



Then, the area is: $A = \frac{1}{2}\text{(base)(height)} = \frac{1}{2}(13)\left(4\sqrt{3}\right) = 26\sqrt{3}\ \text{m}^2$.

g)



In the figure above, by drawing the height of the trapezoid, we got an isosceles right triangle. Hence, $h = \frac{7\sqrt{2}}{\sqrt{2}} = 7$ m. Since the triangle is isosceles, both legs are 7 m. That is, the longer base of the trapezoid is $8 + h = 8 + 7 = 15\ \text{m}$.

Thus, the area is: $A = \frac{1}{2}\left(b_1 + b_2\right)h = \frac{1}{2}(15+8)7 = \frac{161}{2}\ \text{m}^2$.