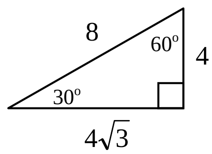
Using Trigonometry to Find Missing Sides of Right Triangles

A. Using a Calculator to Compute Trigonometric Ratios

1. Introduction: Find the following trigonometric ratios by using the definitions of sin(x), cos(x), and tan(x) -- using the mnemonic *SOH-CAH-TOA* -- and then use your calculator to change each fraction to a decimal. (Round your answers to the nearest ten-thousandth, i.e. four decimal places.)



	Trig Ratio		Fraction		Decimal
a)	$\sin(30^\circ)$	=	_______	=	_______
b)	$\cos(30^\circ)$	=	_______	=	_______
c)	$\tan(30^\circ)$	=	_______	=	_______

2. We will now learn how to compute trigonometric ratios with our calculators, using the buttons that look like this:

sin   cos   tan

Note: You must first make sure that your calculator is in degree mode.

To change a TI-83 calculator to degree mode:

a) Press the "Mode" button. You will see a line that says "Radian" and "Degree". (Note: A radian is another unit of angle measurement, in the same way that feet and meters are different units of linear measurement.)

b) If "Degree" is already highlighted in black, then the calculator is already in degree mode. If "Radian" is highlighted, then use the arrow keys on the calculator to highlight "Degree" and then press the "Enter" key. (The "Enter" key is on the bottom right corner of the calculator keypad.)

Every type of calculator is different. Ask your workshop instructor if you need assistance in changing to degree mode.

Example: Now find the following ratios using the [sin] [cos] [tan] buttons on your calculator. Round your answers to the nearest ten-thousandth (four decimal places).

a) $\sin(30^\circ) =$ ___________

For some calculators (including the TI-83), you should first press the "sin" button and then press 30, for other calculators you should press 30 and then press the "sin" button. Ask your workshop instructor if you need assistance.

b) $\cos(30^\circ) =$ ___________

c) $\tan(30^\circ) =$ ___________

Compare your answers from the above exercise to the decimal answers in #1 on the previous page. You should have obtained the same answers. (If not, ask your workshop instructor for assistance.

3. Since we know about the properties of a 30^o-60^o-90^o triangle and know how to easily find the side lengths, we were able to compute the trigonometric ratios for a 30^o angle by hand (in #1 on the previous page). The calculator, however, is able to compute the trigonometric ratios for any angle.

Example: Find the following ratios using the [sin] [cos] [tan] buttons on your calculator. Round your answers to the nearest ten-thousandth (four decimal places).

a) $\sin(47^\circ) =$ ___________

b) $\cos(53^\circ) =$ ___________

c) $\tan(7.5^\circ) =$___________

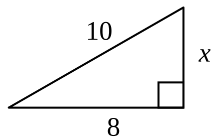
B. Using Trigonometry to Find Missing Sides of Right Triangles

(Note: Figures in this section may not be drawn to scale.)

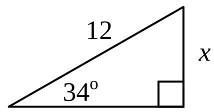
1. Introduction

a) What method can we use to find x in the triangle below?

____



b) Can we use the same method to find x in the following triangle? _______ Why or why not? ____.



2. The word "Trigonometry" can be broken into the following parts:

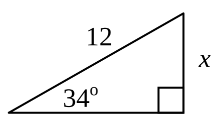
Tri – Gon – Metry, which means

_________ - _________ - ____________________, or in other words:

____

Trigonometry can help us to find measures of sides and angles of right triangles that we were unable to find when our only tool was the Pythagorean Theorem:

3. Let us look again at the following triangle (from #1b above), and learn how to use Trigonometry to find x.



a) Remember that we are using Trigonometry at this point to find missing sides of RIGHT triangles. What other angle (other than the right angle) is given in the picture above? _______. This angle will be our reference angle. (Of course, we could have easily found the other angle of the triangle, and could have used it as our reference angle instead. The answer for x would be the same, but the details of the solution would be different. We will explore this solution later.)

b) Using the 34^{o} angle as your reference angle (i.e. imagine that you are standing at the vertex of that angle), would x be the **opposite leg**, the **adjacent leg**, or the **hypotenuse**? ____________________. Would the 12 be the **opposite leg**, the **adjacent leg**, or the **hypotenuse**? ____________________.

c) Label the triangle above, writing "opposite" (or just "O") beside the x, and writing "hypotenuse" (or just "H") beside the 12. Do not bother to label the "blank" side of the triangle, since we do not know or care to know the length of that side.

d) Now remember our three basic Trigonometric ratios (in abbreviated form):

SOH-CAH-TOA

$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$ $\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ $\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$

Which of the above ratios uses Opposite and Hypotenuse? ________________

e) We know that: $\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$

Remember that θ stands for our reference angle. What is θ? _________
What is the length of the opposite leg? _________
What is the length of the hypotenuse? _________

So, substituting in these values into our trig ratio:

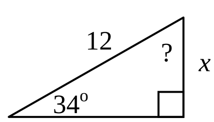
$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$, we obtain

$$\sin(34^\circ) = \frac{x}{12}$$

$$x = 12\sin(34^\circ) \approx 12(0.5592) \approx 6.71$$

So $x \approx 6.71$.

4. Let us look once again at the following triangle (from #1b above), but this time use the other acute angle as the reference angle.



a) What is the measure of the other acute angle of the triangle? _________

b) Now repeat the same process that we followed in #3 above, EXCEPT use the 56° angle as your reference angle. Using the 56° angle as your reference angle, would x be the **opposite leg**, the **adjacent leg**, or the **hypotenuse**? __________________. Would the 12 be the **opposite leg**, the **adjacent leg**, or the **hypotenuse**? ______________________.

c) Label the triangle above, writing "adjacent" (or just "A") beside the *x*, and writing "hypotenuse" (or just "H") beside the 12. Do not bother to label the "blank" side of the triangle, since we do not know or care to know the length of that side.

d) Which of our three Trigonometric ratios (remember *SOH-CAH-TOA*) uses Adjacent and Hypotenuse? ________________

e) We know that: $\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

Remember that θ stands for our reference angle. What is θ? ________
What is the length of the adjacent leg? ________
What is the length of the hypotenuse? ________

So, substituting in these values into our trig ratio:

$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$, we obtain

$$\cos(56^\circ) = \frac{x}{12}$$

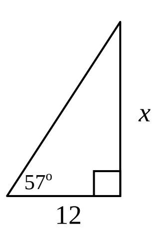
$$x = 12\cos(56^\circ) \approx 12(0.5592) \approx 6.71$$

So $x \approx 6.71$.

Compare this value to our answer from #3. Notice that we obtain the same value for x, regardless of which reference angle we choose to use.
(Note: You may use either acute angle of the triangle as a reference angle, but we never use the 90^o angle as the reference angle.)

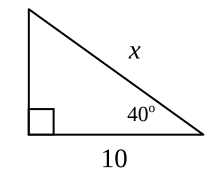
5. Examples: Find *x* in each of the triangles below. Round your final answers to the nearest hundredth. (*Figures may not be drawn to scale.)*

a)



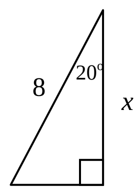
$x \approx$ __________

b)



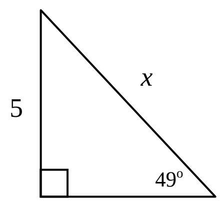
$x \approx$ ____________

c)



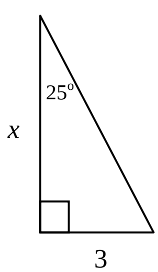
$x \approx$ ____________

d)



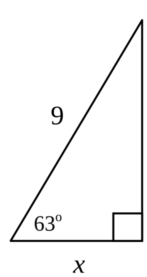
$x \approx$ ____________

e)



$x \approx$ ____________

f)



$x \approx$ ____________