

Using Trigonometry to Find Missing Angles of Right Triangles

A. Using the Inverse Trigonometric Functions on a Calculator

1. Introduction: Consider our methods of solving the following equations:

a) $x + 7 = 10$ *The inverse of addition is subtraction, so we subtract 7 from both sides.*

$$x + 7 - 7 = 10 - 7$$

$$\text{So } x = 3$$

b) $8y = 40$ *The inverse of multiplication is division, so we divide both sides by 8.*

$$\frac{8y}{8} = \frac{40}{8}$$

$$\text{So } y = 5$$

2. Now consider the following equation:

$$\sin(x) = 0.5431$$

We want to isolate x . (Note: We can not divide by “sin” -- it is not a number!) So we need to know the inverse of the sine function.

Look at the buttons on your calculator and see if you can find a key that represents the inverse of the sine function.

On most calculators, this function is labeled “ \sin^{-1} ” can be found in small letters above the “sin” button. (To access this function on most calculators, you need to press another button first, since it is not part of the primary keypad. To access this function on the TI-83 calculator, for example, you need to first press the “2nd” button, and then the “sin” button.)

Back to our example...

$\sin(x) = 0.5431$ Your calculator should still be in degree mode from the previous lesson...

$x = \sin^{-1}(0.5431)$ On the TI-83: Press “2nd”, then “sin”, then 0.5431, then “Enter”.

On some scientific calculators, you first press 0.5431, then “2nd” then “sin”.

$$x \approx 32.89^\circ$$

If you have difficulty obtaining this answer on your calculator, ask your workshop instructor for assistance.

Note: The \sin^{-1} function is NOT a reciprocal function, i.e. $\sin^{-1}(0.5431)$ is NOT the same as

$$\frac{1}{\sin(0.5431)}.$$

3. Use the inverse functions on your calculator to evaluate the following. Round your answers to the nearest hundredth of a degree.

a) $\cos(x) = 0.5431$ $x \approx$ _____

b) $\tan(x) = 0.5431$ $x \approx$ _____

c) $\sin(\theta) = 0.8426$ $\theta \approx$ _____

d) $\tan(B) = 3.5$ $B \approx$ _____

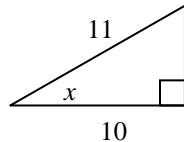
e) $\sin(\theta) = \frac{3}{7}$ $\theta \approx$ _____

f) $\cos(A) = \frac{4}{13}$ $A \approx$ _____

B. Using Trigonometry to Find Missing Angles of Right Triangles

(Note: Figures in this section may not be drawn to scale.)

1. Let us examine the following triangle, and learn how to use Trigonometry to find x .



a) Since x is the angle that we want to find, we will let this angle be our reference angle. Using x as your reference angle (i.e. imagine that you are standing at the vertex of that angle), would the 10 be the **opposite leg**, the **adjacent leg**, or the **hypotenuse**? _____. Would the 11 be the **opposite leg**, the **adjacent leg**, or the **hypotenuse**? _____.

b) Label the triangle above, writing “adjacent” (or just “A”) beside the 10, and writing “hypotenuse” (or just “H”) beside the 11. Do not bother to label the “blank” side of the triangle, since we do not know or care to know the length of that side.

c) Now remember our three basic Trigonometric ratios (in abbreviated form):

SOH-CAH-TOA		
$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$	$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$

Which of the above ratios uses Adjacent and Hypotenuse? _____

d) We know that: $\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

Remember that θ stands for our reference angle. What is θ in our example? _____

What is the length of the adjacent leg? _____

What is the length of the hypotenuse? _____

So, substituting in these values into our trig ratio:

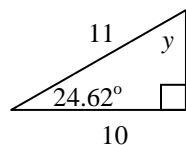
$$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}, \quad \text{we obtain}$$

$$\cos(x) = \frac{10}{11} \quad \text{To isolate } x, \text{ we use the inverse cosine function.}$$

$$x = \cos^{-1}\left(\frac{10}{11}\right) \approx 24.62^\circ$$

$$\text{So } x \approx 24.62^\circ.$$

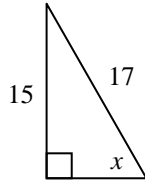
e) What if we now wanted to find the measure of the other acute angle of the triangle? Let us name it y . We certainly could use Trigonometry again, but can you think of a faster way to find the value of y ?



$$y \approx \underline{\hspace{2cm}}$$

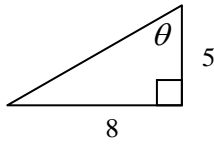
2. Examples: Find the indicated angles in each of the triangles below. Round your final answers to the nearest hundredth. (*Figures may not be drawn to scale.*)

a)



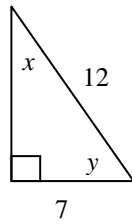
$$x \approx \underline{\hspace{2cm}}$$

b)



$$\theta \approx \underline{\hspace{2cm}}$$

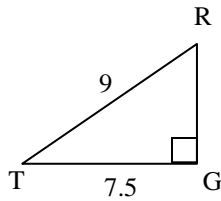
c)



$$x \approx \underline{\hspace{2cm}}$$

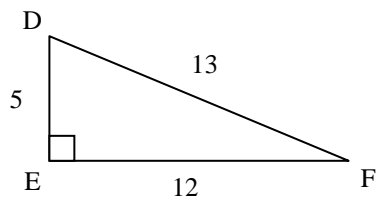
$$y \approx \underline{\hspace{2cm}}$$

d)



$$\angle T \approx \underline{\hspace{2cm}}$$

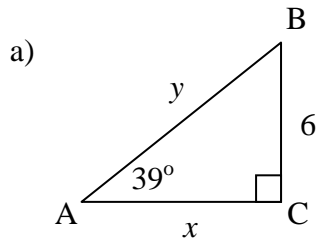
e)



$$\angle D \approx \underline{\hspace{2cm}}$$

$$\angle F \approx \underline{\hspace{2cm}}$$

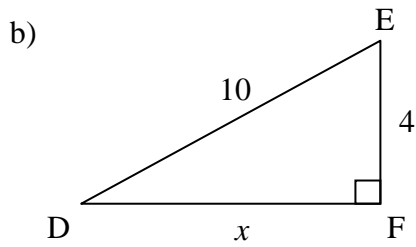
3. Find ALL the missing side and angle measures of the triangles below. Round your answers to the nearest hundredth.



$$x \approx \underline{\hspace{2cm}}$$

$$y \approx \underline{\hspace{2cm}}$$

$$\angle B \approx \underline{\hspace{2cm}}$$



$$x \approx \underline{\hspace{2cm}}$$

$$\angle D \approx \underline{\hspace{2cm}}$$

$$\angle E \approx \underline{\hspace{2cm}}$$