

What's the Rate?

Purpose:

Participants will use rates involving measurement units and dimensional analysis to solve problems.

Overview:

Participants will develop rate cards with a rate involving measurements on one side and the reciprocal of that rate on the other side as a hands-on strategy for solving problems using dimensional analysis. They will select from a set of cards developed from the problem situation to determine which rate(s) to use.

TEKS Mathematics 4-8 Competencies. The beginning teacher:

- III.008.A Selects and uses appropriate units of measurement (e.g., temperature, money, mass, weight, area, capacity, density, percents, speed, acceleration) to quantify, compare, and communicate information.
- III.008.B Develops, justifies, and uses conversions within measurement systems.
- III.008.C Applies dimensional analysis to derive units and formulas in a variety of situations (e.g., rates of change of one variable with respect to another) and to find and evaluate solutions to problems.

TEKS Mathematics Objectives. The student is expected to:

- 5.11B Describe numerical relationships between units of measure within the same measurement system such as an inch is one-twelfth of a foot.
- 6.8D Convert measures within the same measurement system (customary and metric) based on relationships between units.
- 7.3B Estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.

Terms.

Rate, unit rate, reciprocal, dimensional analysis, conversion

Materials.

- Calculators
- Small index cards or pieces of paper the size of small index cards per group of two
- Markers

Transparencies.

- *What's the Rate?*

Activity Sheet(s).

- *What's the Rate?*

Reference.

Johnson, Ken and Herr, Ted(2001). *Problem Solving Strategies: Crossing the River with Dogs* (pp. 249-253). Emeryville, CA: Key Curriculum Press

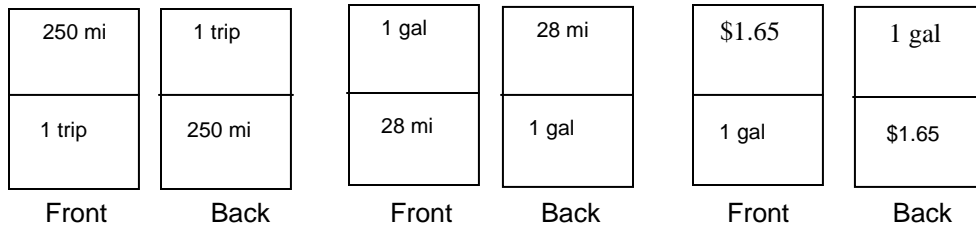
Procedure.

Steps	Questions/Math Notes
<p>1. Have participants read the problem from the transparency <i>What's the Rate?</i> and review the concept of "rate". Remind them that every rate can be expressed in two ways. Example: 3 feet/ yard or 1 yard / 3 feet</p> <p>Inform them that they will write one rate on the front of an index card or paper and the other rate on the back.</p> <p>Model the process of dimensional analysis using the rate cards with an example like the following: <i>Example: How many cups are there in 3 gallons?</i></p> <p>Provide time for participants to work in pairs on their rate cards for the first situation on Activity Sheet <i>What's the Rate?</i> .</p>	<p>Ask participants questions to stimulate their thinking about which rates to use in developing their rate cards.</p> <p><i>What measurement unit(s) are you trying to "keep" in this situation? Explain.</i></p> <p><i>What measurement unit(s) are you trying to "lose" or cancel in the first situation? How do you know?</i></p> <p><i>How does this information help you decide which rate cards to make?</i></p>
<p>2. Provide time for participants to use dimensional analysis to determine "cost of gasoline for the trip" in #1 of Activity Sheet <i>What's the Rate?</i> .</p> <p>Have a few groups write their solution(s) on transparency film and discuss with the whole group.</p>	<p>Dimensional analysis may be a new concept for some middle school teachers. A discussion of the method using cards or pieces of paper with rates on front and back could help them get started. Refer to the solution for examples of how to use the rate cards.</p> <p><i>How did you decide which rates to write on your rate cards?</i></p> <p><i>How did you use your rate cards?</i></p> <p><i>Could you use these rate cards another way? If so, how?</i></p>
<p>3. Have participants work in pairs on numbers 2 through 5 on Activity Sheet <i>What's the Rate?</i>.</p> <p>Monitor their work and ask scaffolding questions when needed to "jump-start" their thinking about the problem.</p> <p>As participants complete a problem, have them record their solution(s) on paper. Ask some groups with different</p>	<p><i>What made you think of these rate cards?</i></p> <p><i>How do you plan to use these cards?</i></p> <p><i>Have you tried any of these cards? What results did these give you?</i></p> <p><i>What is another way to think about the situation?</i></p> <p><i>How do you know that you have completed the problem?</i></p>

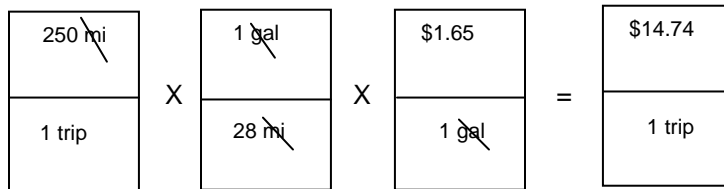
<p>solution(s) to write on transparency film to share with the whole group.</p> <p style="text-align: center;">Steps</p>	<p><i>What is another way that you could solve this problem?</i></p> <p style="text-align: center;">Questions/Math Notes</p>
<p>4. Ask selected groups to discuss their solution(s) on the overhead.</p>	<p><i>Which rate cards did you develop for this problem?</i></p> <p><i>How did you use these cards?</i></p> <p><i>What are some other ways to solve this problem?</i></p>

Solution:

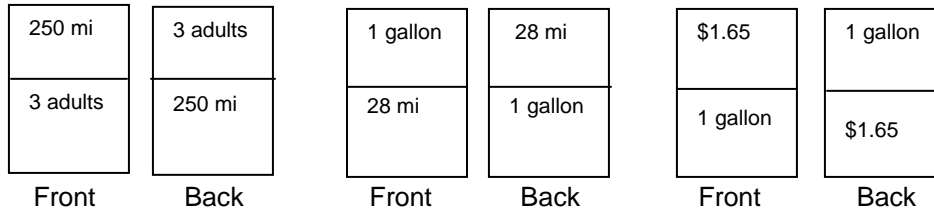
1. Rate Cards for "Cost of gasoline for the trip"



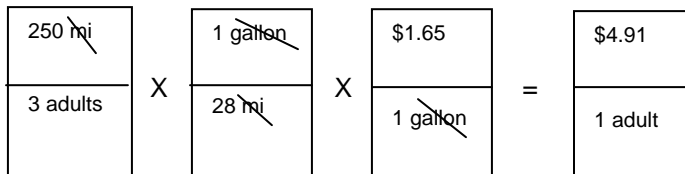
Possible solution:



2. Rate Cards for "Cost in dollars per person"



Possible solution:



3. The solution shown for “speed in feet per second” is one of several possible solutions.

$$\begin{array}{|c|} \hline 55 \text{ mi} \\ \hline 1 \text{ hour} \\ \hline \end{array} \times \begin{array}{|c|} \hline 5280 \text{ ft} \\ \hline 1 \text{ mi} \\ \hline \end{array} \times \begin{array}{|c|} \hline 1 \text{ hour} \\ \hline 60 \text{ min} \\ \hline \end{array} \times \begin{array}{|c|} \hline 1 \text{ min} \\ \hline 60 \text{ sec} \\ \hline \end{array} = \begin{array}{|c|} \hline 80.7 \text{ ft} \\ \hline 1 \text{ sec} \\ \hline \end{array}$$

55 miles per hour is approximately 80.7 feet per sec (rounded to the nearest tenth of foot).

4. Possible solution for “cents per yard traveled”

$$\begin{array}{|c|} \hline 1 \text{ trip} \\ \hline 250 \text{ mi} \\ \hline \end{array} \times \begin{array}{|c|} \hline 1 \text{ mi} \\ \hline 5280 \text{ ft} \\ \hline \end{array} \times \begin{array}{|c|} \hline \$14.74 \\ \hline 1 \text{ trip} \\ \hline \end{array} \times \begin{array}{|c|} \hline 100 \text{ cents} \\ \hline \$1.00 \\ \hline \end{array} \times \begin{array}{|c|} \hline 3 \text{ feet} \\ \hline 1 \text{ yard} \\ \hline \end{array} = \begin{array}{|c|} \hline .0034 \text{ cent} \\ \hline 1 \text{ yard} \\ \hline \end{array}$$

5. Possible solution for “cents per ounce of gasoline”

$$\begin{array}{|c|} \hline \$1.65 \\ \hline 1 \text{ gal} \\ \hline \end{array} \times \begin{array}{|c|} \hline 1 \text{ gal} \\ \hline 4 \text{ qt} \\ \hline \end{array} \times \begin{array}{|c|} \hline 1 \text{ qt} \\ \hline 2 \text{ pt} \\ \hline \end{array} \times \begin{array}{|c|} \hline 1 \text{ pt} \\ \hline 2 \text{ cups} \\ \hline \end{array} \times \begin{array}{|c|} \hline 1 \text{ cup} \\ \hline 8 \text{ oz} \\ \hline \end{array} \times \begin{array}{|c|} \hline 100 \text{ cents} \\ \hline \$1.00 \\ \hline \end{array} \approx \begin{array}{|c|} \hline 1.3 \text{ cents} \\ \hline \text{oz} \\ \hline \end{array}$$