Truly Trulli

Purpose:
Participants will design a net for a cone and construct the cone using the net. They will discuss the geometric concepts and properties involved in designing the net.

Overview:
Participants will be given the dimensions of a cone in a drawing. They will be asked to design a net for the cone and use this net to construct the cone with the given dimensions. Their solution to the problem includes the net, the cone, and a written description of the mathematics used to design the net.

TExES Mathematics 4-8 Competencies. The beginning teacher:

III.008.A Selects and uses appropriate units of measurement (e.g., temperature, money, mass, weight, area, capacity, density, percents, speed, acceleration) to quantify, compare, and communicate information.
III.008.B Develops, justifies, and uses conversions within measurement systems.
III.008.E Applies the Pythagorean theorem, proportional reasoning, and right triangle trigonometry to solve measurement problems.
III.009.A Understands the concepts and properties of points, lines, planes, angles, lengths, and distances.
III.009.B Analyzes and applies the properties of parallel and perpendicular lines.
III.009.C Uses the properties of congruent triangles to explore geometric relationships and prove theorems.
III.010.A Uses and understands the development of formulas to find lengths, perimeters, areas, and volumes of basic geometric figures.
III.010.B Applies relationships among similar figures, scale and proportion and analyzes how changes in scale affect area and volume measurements.
III.010.C Uses a variety of representations (e.g., numeric, verbal, graphic, symbolic) to analyze and solve problems involving two-and three-dimensional figures such as circles, triangles, polygons, cylinders, prisms, and spheres.
III.010.D Analyzes the relationship among three-dimensional figures and related two-dimensional representations (e.g., projections, cross-sections, nets) and uses these representations to solve problems.

TEKS Mathematics Objectives. The student is expected to:

4.9.A Demonstrate translations, reflections, and rotations using concrete models.
4.9.B Use translations, reflections, and rotations to verify that two shapes are congruent.
5.8A Sketch the results of translations, rotations, and reflections.
7.8.B Make a net (two-dimensional model) of the surface area of a solid.
8.8.A Find surface area of prisms and cylinders using concrete models and nets (two-dimensional models)

Materials.
- 11” x 17” paper or chart paper for making nets
- Scissors
- Tape
- Protractors
- Compasses
- Computer projection device (optional)
- Computer with internet capabilities (optional)

Terms.
Cone, slant height, radius, circle, height or altitude of cone, circumference, pi, Pythagorean theorem, ratio, proportion, right triangle, base of cone, lateral surface, arc length, central angle, sector of a circle, ray, straight angle

Transparencies.
- *Truly Trulli*

Activity Sheet(s).
- *Truly Trulli*

References:
TEXTEAMS Rethinking Middle School Mathematics: Geometry Across the TEKS (middle school)(2001). Austin, TX: The Charles A. Dana Center

http://www.geocities.com/trullihouses/trulli3.htm

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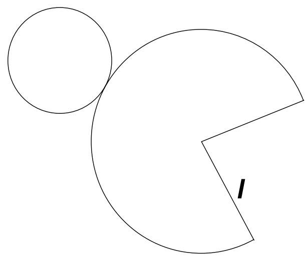
Procedure:

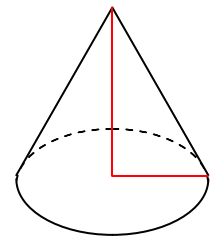
Steps	Questions/Math Notes
1. Place the Transparency *Truly Trulli* on the overhead and have participants read the introduction about the trulli houses located in Italy. Ask if anyone has ever visited this historic site to view these unusual structures with the cone-shaped roofs. If a computer with internet capabilities and a projection device are available, a visit to the website listed in the transparency can provide a meaningful introduction through pictures of the trulli. A trip to a computer lab with internet capabilities would provide a "get-up-and-move activity and create interest in the problem. The cone-shaped roofs are said to create a thermos effect making the homes cool in the summer and warm in the winter. The thick stone walls also affect the cooling and heating. Inform participants that their work with the trulli involves right circular cones and that when you use the word "cone" in this problem, you are referring to a right circular cone.	Ask participants questions to engage them in the problem and extend their thinking about cones and how to design a net for a cone. *What would be the benefit of building a house of stone such as the trulli?* *Why do you think these homes were built in this manner?* *How would the cone-shaped roofs affect the size of the trulli homes?* *How do you think these cone-shaped roofs affect the heating and cooling of these homes?*

2. Have participants make a sketch of a net for a cone and review the properties of a right circular cone. Participants may need to be reminded that a net is a two-dimensional pattern for a solid. They may also need to review the following concepts: a) Pythagorean theorem b) Finding the square root of a number c) Finding arc length d) Using a protractor e) Finding the circumference of a circle	*What do all of your nets have in common?* *How many different nets for a cone are possible? Explain.* *How would you describe the altitude of a right circular cone? The base?* *How does the altitude compare with the slant height of a right circular cone?* *What do you know about the part of your net that looks like a slice of pie?* *How would you describe the edge of your net that wraps around the circular base?*
3. Have participants work in pairs on Activity Sheet *Truly Trulli.* Recommend that they leave a tab on one side of their sector so that it is easier to secure the edges to make the cone. Monitor their work as they make their nets and cones and ask them questions about the mathematics involved.	*What is the relationship between the arc length of the sector and the circular base?* *What is the relationship between the radius of the sector and your cone?* *How can you determine the degrees of a sector?* *What is the relationship between the degrees of a sector and the arc degrees?* *How can you find the length of the arc in your sector?* *How can this information help you solve the problem?*
4. Debrief the activity by having several groups explain how they designed their nets using geometric concepts and properties.	*How did you find the slant height of the cone?* *How did the slant height help you solve this problem?* *How did you determine the degrees in the central angle that forms the sector in your net?* *What made you think to write that equation?* *Did anyone write a different equation? Come up and explain how your equation relates to the problem.* *What seemed to present the most challenge in making a net for your cone? Explain.* *How has making a net for a cone helped you better understand geometric concepts and properties?*

Possible solution: (scale: 1 in. in the net represents 2 ft. in the actual cupola)

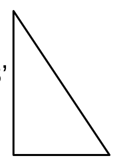


h = 8 ft
r = 4 ft.



The slant height ***l*** of the cupola is the radius of the sector whose arc length wraps around the circular base with radius ***r*.** Therefore, find the slant height of the cupola using the Pythagorean theorem as follows.

h = 8'



r = 4'

$l^2 = r^2 + h^2$
$l^2 = 4^2 + 8^2$
$l^2 = 16 + 64$
$l^2 = 80$
$l = \sqrt{80}$
$l = 4\sqrt{5}$ ft.or approximately 9 ft.

The arc length of the sector wraps around the circumference of the circular base. Find the circumference of the circular base and the arc length of the sector as follows.

Circumference of circular base:

$C = 2\pi r$
$C = 2\pi(4)$
$C = 8\pi$ ft.

Arc length of sector:
n represents the arc degrees

Arc length = $(n/360)(2\pi r)$
Arc length = $(n/360)(2\pi)(l)$
Arc length = $\pi n l / 180$ ft.
Arc length = $\pi n(4\sqrt{5}) / 180$ ft
Arc length = $\pi n(\sqrt{5}) / 45$ ft.

Since the arc length has the same measure as the circumference of the circular base, the following equation can be written and solved to find the degrees of the arc. The central angle that intercepts the arc will have the same degree measure.

$8\pi = \pi n(\sqrt{5}) / 45$
$8 = n(\sqrt{5}) / 45$
$8(45) / \sqrt{5} = n$
$161° \approx n$

To design the net for the cupola, start by constructing the circular base using a compass and a radius of 2 inches (1 inch in the net represents 2 feet in the actual cupola). The radius of the sector that wraps around the circular base of the cupola is approximately 9 ft. and represents the slant height of the cupola. Use a radius of 4.5 inches and touch the circular base with the tip of the pencil end of the compass. This point will represent the point of tangency of the sector and the circular base to make one piece for the net. Make an arc of 180°. There are now two opposite rays that make this straight angle. Use a protractor to measure an angle of 161° to complete the sector. Allow a margin of approximately ¼ inch along one radius of the sector to make it easier to form the cone. Use tape to secure the cone. You now have a model for the cupola of a “Truly Trulli”!