Glass Wrap

Purpose:
Participants will design a net for a pyramid with a square base that is a replica of the luxurious Luxor Hotel in Las Vegas. They will determine the number of square feet of glass that is needed to "wrap" the Luxor Hotel in glass on all its lateral faces.

Overview:
Participants will construct a replica of the luxurious pyramid-shaped Luxor Hotel in Las Vegas by designing a net using a scale of their choice. They will determine how many square feet of glass it would take to "wrap" the Luxor Hotel on all its lateral faces with glass and compare to the number of actual square feet of glass that was used on the lateral surface of the structure. Then they will convert the actual number of square feet of glass to acres of glass for the "glass wrap" of the Luxor Hotel and compare to the actual acres of glass used in the structure.

TExES Mathematics 4-8 Competencies. The beginning teacher:

III.008.A Selects and uses appropriate units of measurement (e.g., temperature, money, mass, weight, area, capacity, density, percents, speed, acceleration) to quantify, compare, and communicate information.
III.008.B Develops, justifies, and uses conversions within measurement systems.
III.008.E Applies the Pythagorean theorem, proportional reasoning, and right triangle trigonometry to solve measurement problems.
III.009.A Understands the concepts and properties of points, lines, planes, angles, lengths, and distances.
III.009.B Analyzes and applies the properties of parallel and perpendicular lines.
III.009.C Uses the properties of congruent triangles to explore geometric relationships and prove theorems.
III.010.A Uses and understands the development of formulas to find lengths, perimeters, areas, and volumes of basic geometric figures.
III.010.B Applies relationships among similar figures, scale and proportion and analyzes how changes in scale affect area and volume measurements.
III.010.C Uses a variety of representations (e.g., numeric, verbal, graphic, symbolic) to analyze and solve problems involving two-and three-dimensional figures such as circles, triangles, polygons, cylinders, prisms, and spheres.
III.010.D Analyzes the relationship among three-dimensional figures and related two-dimensional representations (e.g., projections, cross-sections, nets) and uses these representations to solve problems.

TEKS Mathematics Objectives. The student is expected to:

4.9.A Demonstrate translations, reflections, and rotations using concrete models.
4.9.B Use translations, reflections, and rotations to verify that two shapes are congruent.
5.8A Sketch the results of translations, rotations, and reflections.
7.8.B Make a net (two-dimensional model) of the surface area of a solid.
8.8.A Find surface area of prisms and cylinders using concrete models and nets (two-dimensional models)

Materials.

- Black poster board (low to medium weight)
- Scissors
- Centimeter grid paper
- Tape

Terms.

Pyramid, net, triangle, perpendicular, altitude, slant height, lateral face, base, surface area, apex, vertex, square, diagonal,area, lateral surface area, acre

Transparencies.

- *Glass Wrap*

Activity Sheet(s).

- *Glass Wrap*

References:

www.mathprojects.com/downloads/geometry/luxor.pdf

http://www.travel2vegas.com/hotel/luxorhotel.html

TEXTEAMS Rethinking Middle School Mathematics: Geometry Across the TEKS (middle school)(2001). Austin, TX: The Charles A. Dana Center.

Procedure:

Steps	Questions/Math Notes
1. Put the Transparency *Glass Wrap* on the overhead and ask participants if anyone has visited the famous Luxor Hotel in Las Vegas. A picture of the hotel can be seen online at http://www.vegas.com/resorts/luxor . This discussion can serve as an interesting introduction to nets of pyramids. Ask participants to explain what they are to do in the problem in their own words. Ask them questions about their understanding of the properties of a right pyramid. It is especially important to distinguish between the height of the pyramid and its slant height. Inform them that one acre is 43, 560 square feet.	Ask participants questions to spark their interest in creating nets and to make connections among geometric concepts related to this problem. *How would you describe the geometric properties of the Luxor Hotel in Las Vegas?* *What type of lateral faces does the Luxor Hotel have?* *How would you describe the dimensions of each lateral face?* *What does an altitude of 350 feet in the structure mean?* *How does a slant height differ from the height of the pyramid?* *How would you describe a net for any geometric solid?* *What additional information do you need to solve this problem?*
2. Have participants work in small groups to make a net for their replica of the Luxor Hotel. After they design a net, they can trace the net onto black poster board. The net can then be folded into a square pyramid. Participants are to answer question 2 on the Activity Sheet *Glass Wrap*.	*What made you think to select that particular scale for your model?* *What do you know about the relationship between the altitude/height of the pyramid and the square base?* *Is it possible to make a different net for a square pyramid? Explain.*

Monitor their work and select a few groups with different solution strategies to write their solutions to the problem on an overhead transparency.	
3. When all groups have designed a replica of the Luxor Hotel, have them place their models with the net designed and supporting documentation in the middle of their table and conduct a gallery tour. Have participants compare and contrast their own work/solution with the other groups' solutions. After the gallery tour, debrief the activity making connections among key concepts in geometry.	*How did your net for a square pyramid compare with those of other groups?* *What do all of the nets have in common?* *How do you think the perimeters of these different nets compare? Let's look at some to verify your conjecture.* *How did you determine the area of each face of the Luxor Hotel?* *How did you convert square feet into acres?* *How does the number of acres you determined for the "glass wrap" of the Luxor Hotel compare with the actual number of acres of glass used on the exterior of the hotel?* *What do you think accounts for the difference?* *What important geometric concepts did you use to solve this problem?*

Possible solution:

1. For this sample solution, 1 cm in the model represents 40 ft in the actual hotel.

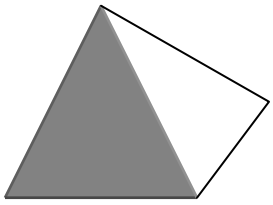


figure 1

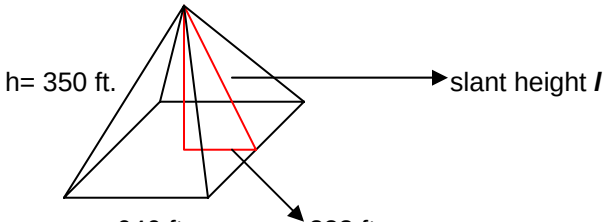
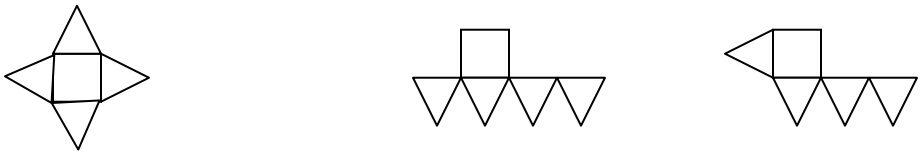


figure 2

There are a variety of nets that can be used to form the replica of the Luxor Hotel. A few are shown below:

Net 1 Net 2 Net 3



Using the scale of 1 cm in the model represents 40 ft. in the actual hotel, the dimensions of the scale model will be as follows:

Square base dimensions: side lengths of approximately 16.2 cm
646 ft. x 1 cm/40 ft. ≈ 16.2 cm

Altitude: 8.75 cm
350 ft. x 1 cm/40 ft.= 8.75 cm

Slant height: 11.9 cm
476.3 ft. x 1 cm/40 ft ≈ 11.9 cm

Procedure used to design Net 1: You may need to tape sheet(s) of centimeter grid paper together to get the net on 1 large sheet of paper.

Construct a square on centimeter grid paper with sides about 16.2 cm in length. Construct the perpendicular bisector of each side of the square or use paper folding to determine the perpendicular bisector. Mark off a distance of 8.75 cm from the midpoint of each side along the perpendicular bisector of each side. This distance (8.75 cm) represents the slant height of each lateral face of the pyramid and the altitude of each triangular face. Each of these segments has the midpoint of a side as an endpoint and the apex of the pyramid as the other endpoint. Connect the apex to the endpoints of each side of the square to form a set of 4 triangles as shown in figure 1. Each of these isosceles triangles will share a base in common with a side of the square. Cut out the entire net and fold to form a replica of the Luxor Hotel in Las Vegas. Use tape to secure the edges.

Procedure used to design Net 2:
Construct a square on centimeter grid paper with sides about 16.2 cm. Construct the perpendicular bisector of one side of the square or use paper folding to determine the perpendicular bisector. Mark off a distance of 8.75 cm from the midpoint of the side along the perpendicular bisector of that side. Connect the apex to the endpoints of the segment to form an isosceles triangle. Translate the triangle as shown in figure 2 to complete the net. Cut out the net, fold, and tape to complete a replica of the Luxor Hotel.

Procedure used to design Net 3:
Construct a square on centimeter grid paper with sides about 16.2 cm. Construct the perpendicular bisector of one side of the square or use paper folding to determine the perpendicular bisector. Mark off a distance of 8.75 cm from the midpoint of the side along the perpendicular bisector of that side. Connect the apex to the endpoints of the segment to form an isosceles triangle. Use one rotation and translations to complete the net as shown in figure 3. Cut out the net, fold, and tape to complete a replica of the Luxor Hotel.

2. Note the red triangle in figure 2. The altitude is drawn from the apex of the pyramid to the point where the diagonals of the square base intersect. This point of intersection is equidistant from the sides of the square base. The legs of the right triangle (in red) are 350 ft. and 323 ft. The Pythagorean theorem can be used to find the slant height of one lateral face of the pyramid:

$$350^2 + 323^2 = l^2$$
$$122{,}500 + 104{,}329 = l^2$$
$$226{,}829 = l^2$$
$$476.3 \text{ ft.} \approx l$$

Using the slant height of one of the congruent faces of the Luxor Hotel, the area can be determined for one of the triangular faces.

$$A = \tfrac{1}{2}\, bh$$
$$A = 0.5 (646)(476.3)$$
$$A \approx 153{,}844.9 \text{ sq. ft.}$$

Since there are four congruent triangular faces of the Luxor Hotel, multiply 153,844.9 sq. ft by 4 and get the lateral surface area of 615,379.6 sq. ft. We are given that there are 43,560 sq. ft. in 1 acre. By using dimensional analysis, the number of acres for the lateral surface area can be determined.

$$153{,}844.9 \text{ sq. ft.} \times 1 \text{ acre}/43{,}560 \text{ sq. ft} \approx 14.1 \text{ acres}$$

The number 14.1 represents the number of acres of glass that it would take to "glass wrap" the exterior of the Luxor Hotel. But, we are told that 13 acres of glass were used on the exterior of this hotel. What could account for the difference? Perhaps the difference is accounted for in the steel beams that make up the edges and joints. Answers to this question will vary.

Find the lateral surface area of the replica and compare to that of the actual lateral surface area of the Luxor Hotel.

Dimensions of replica: h= 8.75 cm, side length of square base = 16.2 cm, slant height=11.9 cm

Area of one of the lateral faces:

$$A = \tfrac{1}{2}\, bh$$
$$A = 0.5 (16.2)(11.9)$$
$$A = 96.4 \text{ sq. cm}$$

Multiply 96.4 sq. cm by 4 to get the lateral surface area of the replica: 385.6 sq. cm

Compare the surface area of the Luxor Hotel to the replica: