Super Size or Down Size?

Purpose:
Participants will investigate the effects of a scale factor change on the volume and surface area of a solid geometric figure. They will determine the relationship between a change in dimensions by a given scale factor and the resulting change in volume and surface area.

Objective:
Participants will use cubes to build models of rectangular prisms and investigate the effect of a change in dimensions by a given scale factor on the resulting volumes of these three-dimensional figures. They will also consider the effect of a scale factor change in dimensions on the surface area of these prisms. After recording the data and studying patterns in a table, participants will make conjectures, state generalizations, and make connections among the different representations (concrete model, table, and symbolic rule).

TExES Mathematics 4-8 Competencies. The beginning teacher:

III.008.A Selects and uses appropriate units of measurement (e.g., temperature, money, mass, weight, area, capacity, density, percents, speed, acceleration) to quantify, compare, and communicate information.

III.010.A Uses and understands the development of formulas to find lengths, perimeters, areas, and volumes of basic geometric figures.

III.010.B Applies relationships among similar figures, scale, and proportion and analyzes how changes in scale affect area and volume measurements.

III.010.C Uses a variety of representations (e.g., numeric, verbal, graphic, symbolic) to analyze and solve problems involving two-and three-dimensional figures such as circles, triangles, polygons, cylinders, prisms, and spheres.

V.016.D Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphic, pictorial, symbolic, concrete).

V.016.E Demonstrates an understanding of the use of visual media such as graphs, tables, diagrams, and animations to communicate mathematical information.

TEKS Mathematics Objectives. The student is expected to:

4.14.D Use tools such as real objects, manipulatives, and technology to solve problems.

5.10.A Measure volume using concrete models of cubic units.

5.10.B Estimate volume in cubic units.

5.14.D Use tools such as real objects, manipulatives, and technology to solve problems.

6.11.D Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.

7.9 Estimate measurements and solve application problems involving length (including perimeter and circumference), area, and volume.

7.13.D Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.

8.8.B Connect models to formulas for volume of prisms, cylinders, pyramids, and cones.

8.8C Estimate answers and use formulas to solve application problems involving surface area and volume.

8.14.D Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.

Terms.
Prism, cube, scale factor, volume, conjecture, generalization, units of measure, ratio, proportion, rectangular prism, cylinder, surface area

Materials.
- One-inch cubes
- One-inch grid/graph paper
- Color tiles
- Markers
- Graphing calculators

Transparencies.
- *Super Size or Down Size?*

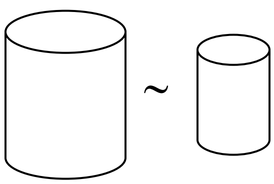
Activity Sheet(s).
- *Super Size or Down Size?*

Procedure:

Steps	Questions/Math Notes
1.Have participants read the problem on the Transparency *Super Size or Down Size?.* Have them work in small groups of 2 to 4 on the investigations presented in Activity Sheet *Super Size or Down Size?*. They will need 1" cubes for the volume investigations and color tiles and/or 1" grid paper for the surface area investigations. Explain to participants that they are to focus on process using the manipulatives. They will be expected to articulate connections between and among the different representations (verbal, concrete, tabular, and symbolic). The dimensions of the rectangular prisms in the *"possible solution"* could be given to participants. The other option is to have them create their own.	Ask participants questions to stimulate their thinking about the effect of a scale factor change in the dimensions of a geometric solid on the resulting surface area and volume of that figure. *While many middle school teachers may know this relationship, they may not know how to model this with students using a variety of representations. Their ability to model and articulate the process will help to clarify their thinking about this significant concept in the TExES Mathematics 4-8 Competencies(III.010.B) and the TEKS.*
2.Monitor the work of participants and ask questions to clarify their thinking and keep them focused on process.	*What is the relationship between the volume of your original geometric figure and the volume of that figure when the dimensions were doubled?* *Show me in your model. Where can that be seen in your process column? What does it mean?* *What is the relationship between the surface area of your original geometric figure and the surface area of that figure with a scale factor change of ½ the original dimensions?* *Where can I find this in your table? In your rule? Explain.*

3. Have several groups present their findings on transparency film to the whole group. Ask them to model with the manipulatives as needed to make connections between and among the different representations.	*What generalization(s) did you articulate for your investigation(s) with volume?* *Could you extend the generalization(s) to any prism? Explain.* *Describe a scale factor "K" that would produce a "super size" in volume. How would you describe a scale factor that would produce a "down size" in volume?* *Suppose you only double one dimension of a rectangular prism. How would this affect your rule for surface area of that figure? Your rule for volume of that figure?* *How could you apply your generalizations to other geometric figures such as cylinders and spheres? Explain.*

Extension:
Two right circular cylinders are similar with the information given below.



The ratio of their volumes is 512:125.

The area of one circular base of the smaller cylinder is 75π cm^2.
The height of the smaller cylinder is 10 cm.

Find the actual volume and surface area of the larger cylinder.
Solution:
Since the ratio of the volumes of the two cylinders is 512:125 and the cylinders are similar, the ratio of corresponding parts (radii, diameters, heights) is the cube root of 512:125. Therefore, the ratio of the corresponding radii is 8:5. The area of the base of the smaller cylinder is 75π cm^2. By solving the equation, $\pi r^2 = 75\pi$, $r = 5\sqrt{3}$ cm. Let r_1 represent the radius of the larger cylinder and solve the proportion $8:5 = r_1 : 5\sqrt{3}$ for r_1 and get $\mathbf{r_1 = 8\sqrt{3}}$. The ratio of corresponding heights h_1:h is also 8:5. Solve the equation $h_1:10 = 8:5$ for h_1 and get **h_1 = 16 cm.** Using r_1 and h_1, the actual surface area and volume of the larger cylinder can be determined.

Surface Area = $2\pi rh + 2B$ where B represents the area of a circular base
$= 2\pi(8\sqrt{3})(16) + 2(\pi)(8\sqrt{3})^2$
$= (256\sqrt{3} + 384)$ sq cm

Volume $= Bh$
$= (192\pi)(16)$
$= 3072\pi$ cu cm

Solution:

Table 1: Investigations with Volume (Super Size)

Sketch of 3-D Figure	Sketch of 3-D Figure with dimensions doubled	Volume of Original Figure Process	Volume when dimensions are doubled Process	Volume of Original Figure	Volume when dimensions are doubled
		L x W x H 1 x 1 x 1	**L x W x H** 2 x 2 x 2 8 groups of 1 in^3 2^3x1 cu. in.	1 cu. in.	8 cu. in.
		2 x 1 x 1	4 x 2 x 2 8 groups of 2 in^3 2^3x2 cu. in.	2 cu. in.	16 cu. in.
		3 x 2 x 1	6 x 4 x 2 8 groups of 6 in^3 2^3x6 cu. in.	6 cu. in.	48 cu. in.

Generalization: When the dimensions of a rectangular prism are increased by a scale factor of 2, the resulting volume is 2^3 times the original volume.

Table 2: Investigations with Volume (Down Size)

Sketch of 3-D Figure	Sketch of 3-D Figure with dimensions halved	Volume of Original Figure Process	Volume when dimensions are halved Process	Volume of Original Figure Process	Volume when dimensions are halved Process
		L x W x H 6 x 4 x 2	**L x W x H** 3 x 2 x 1	48 cu. in. 8 groups of 6 $in.^3$ 2^3x6 cu. in.	6 cu. in. 1/8 x 48 in^3 $(1/2)^3$ x 48 in^3
		4 x 2 x 2	2 x 1 x 1	16 cu. in. 8 groups of 2 $in.^3$ 2^3x2 cu. in.	2 cu. in. 1/8 x 2 $in.^3$ $(1/2)^3$ x 2 in^3
		4 x 6 x 4	2 x 3 x 2	96 cu. in. 8 groups of 12 $in.^3$ 2^3x12 cu. in.	12 cu. in. 1/8 x 96 $in.^3$ $(1/2)^3$ x 12 in^3

Generalization: When the dimensions of a rectangular prism are changed by a scale factor of ½, the resulting volume is $(1/2)^3$ times the original volume.

Table 3: Investigations with Surface Area (Super Size)

Dimensions of original figure	Dimensions of figure with a scale factor of 2	Surface Area of original figure **Process**	Surface Area when dimensions are doubled **Process**	Surface Area of Original Figure	Surface Area when dimensions are doubled
L x W x H 1 x 1 x 1	**L x W x H** 2 x 2 x 2	**SA= 2LW+2LH+2WH** 2(1)+2(1)+2(1)	**SA= 2LW+2LH+2WH** 2(4)+2(4)+2(4)	6 sq. in.	24 sq. in. 4 groups of 6 in^2 2^2 x 6 sq. in.
2 x 2 x 1	4 x 4 x 2	2(4)+2(2)+2(2)	2(16)+2(8)+2(8)	16 sq. in.	64 sq. in. 4 groups of 16 in^2 2^2 x 16 sq. in.
3 x 2 x 1	6 x 4 x 2	2(6)+2(3)+2(2)	2(24)+2(12)+2(8)	22 sq. in.	88 sq. in. 4 groups of 22 in^2 2^2 x 22 sq. in.

Generalization: When the dimensions of a rectangular prism are changed by a scale factor of 2, the surface area of the figure is changed by a scale factor of 2^2.

Table 4: Investigations with Surface Area (Down Size)

Dimensions of original figure	Dimensions of figure with a scale factor of 1/2	Surface Area of original figure **Process**	Surface Area when dimensions are halved **Process**	Surface Area of Original Figure	Surface Area when dimensions are halved
L x W x H 6 x 4 x 2	**L x W x H** 3 x 2 x 1	**SA= 2LW+2LH+2WH** 2(24)+2(12)+2(8)	**SA= 2LW+2LH+2WH** 2(6)+2(3)+2(2)	88 sq. in.	22 sq. in. ¼ x 88 sq. in. $(1/2)^2$ x 88 $in.^2$
4 x 2 x 2	2 x 1 x 1	2(8)+2(8)+2(4)	2(2)+2(2)+2(1)	40 sq. in.	10 sq. in. 1/4x40 sq. in. $(1/2)^2$ x 40 $in.^2$
4 x 6 x 4	2 x 3 x 2	2(24)+2(16)+2(24)	2(6)+2(4)+2(6)	128 sq. in.	32 sq. in. 1/4x128 sq. in. $(1/2)^2$ x 128 $in.^2$

Generalization: When the dimensions of a rectangular prism are changed by a scale factor of ½, the surface area is changed by a scale factor of $(1/2)^2$.