Without a Shadow of a Doubt!

Purpose:
Participants will use right triangle trigonometry and indirect measurement to determine the height of a tall object such as a flagpole, building, or tree as an application of geometry in the real-world. Similar triangles can be used as an alternate solution strategy.

Overview:
Participants will solve the problem situation "*Without a Shadow of a Doubt*!" and discuss the geometry concepts involved. They will "mirror" the experiment presented in the problem to determine the height of a tall object such as a flagpole, building, or tree using indirect measurement and right triangle trigonometry. Similar triangle properties can be used as a second solution strategy.

TExES Mathematics 4-8 Competencies. The beginning teacher:
III.008.B Develops, justifies, and uses conversions within measurement systems.
III.008.D Describes the precision of measurement and the effects of error on measurement.
III.008.E Applies the Pythagorean theorem, proportional reasoning, and right triangle trigonometry to solve measurement problems.
III.009.A Understands concepts and properties of points, lines, planes, angles, lengths, and distances.
III.009.B Analyzes and applies the properties of parallel and perpendicular lines.
III.010.B Applies relationships among similar figures, scale, an proportion and analyzes how changes in scale affect area and volume measurements.
III.010.C Uses a variety of representations (e.g., numeric, verbal, graphic, symbolic) to analyze and solve problems involving two-and three-dimensional figures such as circles, triangles, polygons, cylinders, prisms, and spheres.

TEKS Mathematics Objectives. The student is expected to:
4.8A Identify right, acute, and obtuse angles.
4.8B Identify models of parallel and perpendicular lines.
4.12 Measure to solve problems involving length, including perimeter, time, temperature, and area.
5.7A Identify critical attributes including parallel, perpendicular, and congruent parts of geometric shapes and solids.
5.7B Use critical attributes to define geometric shapes or solids.
5.11A Measure to solve problems involving length (including perimeter), weight, capacity, time, temperature, and area.
5.11B Describe numerical relationships between units of measure within the same measurement system such as an inch is one-twelfth of a foot.
6.3A Use ratios to describe proportional situations.
6.3C Use ratios to make predictions in proportional situations.
6.6A Use angle measurements to classify angles as acute, obtuse, or right.
6.6B Identify relationships involving angles in triangles and quadrilaterals.
6.8B Select and use appropriate units, tools, or formulas to measure and to solve problems involving length (including perimeter and circumference), area, time, temperature, capacity, and weight.
7.2D Use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio.
7.6B Use properties to classify shapes including triangles, quadrilaterals, pentagons, and circles.
7.6D Use critical attributes to define similarity.
8.1B Select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships.
8.9B Use proportional relationships in similar shapes to find missing measurements.

Materials.

- Mylar mirrors (4” x 4”) from a crafts store
- Measuring tape (50’ to 100’)

Transparencies.

- *Without a Shadow of a Doubt!*

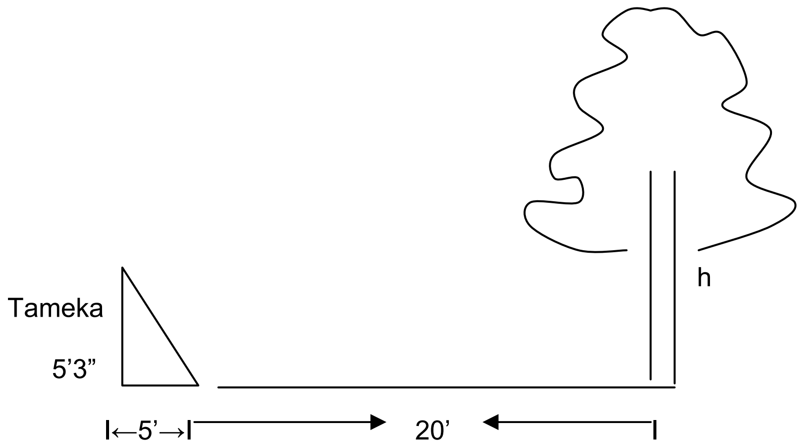
Activity Sheet(s).

- *Without a Shadow of a Doubt!*

Procedure:

Steps	Questions/Math Notes
1. Have participants read the problem on the Transparency *Without a Shadow of a Doubt!.* Ask them to make a sketch of the situation in the problem and make a list of what they know and what they need to know.	Ask questions to check for understanding of the problem and the nature of the experiment they will be doing in groups of 2 to 4. *What do you know in this problem? What do you need to know?* *Which angle represents the angle of incidence in your drawing/sketch? The angle of reflection?* *How are these angles formed?* *How would you describe the triangles formed?* *What do these triangles have in common?* *How are they different?*
2. Have participants work in pairs on a solution to Tameka’s math/science project. Ask several groups to share their solution(s) to the problem on an overhead transparency.	*How did you use the angle of incidence and angle of reflection to help you solve this problem?* *How can you use right triangle trigonometry to solve this problem?* *What are some other method(s) you could use to find the height of the tree?*
3. Have participants work in small groups of 2 to 4 to “mirror” the experiment conducted by Juan and Tameka to determine the height of a tall object such as a flagpole, building, or tree. This can be a fun “get-up-and-move” outdoor activity with a real-world connection. The groups are to use the Activity Sheet *Without a Shadow of a Doubt!*, a mirror, and a tape measure. Have them make a sketch of their problem situation and show how they determined the height of the tall object.	*What geometry concepts did you apply in your investigation?* *What other key mathematics concepts were involved in the solution of your problem?* *How could you apply these same concepts in other problem situations?*

Debrief the activity after participants have answered the questions on the Activity Sheet *Without a Shadow of a Doubt!* .	



Possible solution. Tameka's problem: 21 feet

Solution Strategy Using Right Triangle Trigonometry:
5'3" = 5.25 feet

tan θ = 5.25/5 = 1.05
$\theta \approx 46.4°$ (rounded to the nearest tenth of a degree)
Since the angle of incidence (θ) is congruent to the angle of reflection (α), these angles have the same measure. The measure of angle α equals the measure of angle $\theta \approx 46.4°$

tan 46.4 = h/20
20 tan 46.4 = h
20(1.05) = h
21 feet = h

1. Refer to #1 in Similar Triangle Solution.
2. The two triangles are right triangles. The tangent ratio in right triangle trigonometry can be applied in this problem because the lengths of two legs in the smaller right triangle are known.
3. The tangent ratio (length of side opposite/length of side adjacent) in the larger right triangle can be used to determine *h*, the height of the tree. Math concepts that can be used to solve this problem include the following:
    a. Perpendiculars form right angles
    b. All right angles are congruent
    c. A triangle with one right angle is a right triangle
    d. Tangent ratio for right triangles
    e. Ratios
    f. Solving an equation
    g. Rounding to the nearest tenth
    h. Congruent angles
    i. Congruent angles have the same measure

Solution Strategy Using Similar Triangles:

5 feet 3 inches = 5.25 feet

5.25 / 5 = h / 20

Using equivalent ratios or a scale factor of 4, 5x4=20 and 5.25x4 = 21.

The height of the oak tree in Tameka's problem is 21 feet based upon the measurements determined on site at the neighborhood park.

Answers will vary by group based upon how each group set up the problem on site.

Answers to questions on Activity Sheet # :

1. The "angle of incidence" is the angle formed by a ray of light and the horizontal mirror; the "angle of reflection" is formed by the reflection of that ray of light and the horizontal mirror.

2. The two triangles in the drawing are similar by AAA: right angles are congruent, the angle of incidence has the same measure as the angle of reflection, and the other acute angles are congruent by complements of congruent angles are congruent.

3. Since the triangles are similar, their corresponding sides (opposite corresponding angles) are in proportion. Equivalent ratios or scale factors can be used to solve proportions in the problem. Key concepts used in this problem include the following:

    a. Triangles can be similar by AAA.
    b. A right triangle has one right angle.
    c. The acute angles of a right triangle are complementary.
    d. Complements of congruent angles are congruent.
    e. Corresponding sides of similar triangles are proportional.
    f. A proportion can be solved by equivalent ratios, scale factors, or equations.
    g. Measurements of the same object using different units of measure are proportional.
    h. Perpendiculars form right angles.
    i. All right angles are congruent.
    j. Congruent angles have the same measure.