The Big Roll

Purpose:
Participants will find the thickness of one sheet on a "big roll" of paper towels using direct and indirect measurement. They will also determine the percent of error in their findings based upon the thickness data provided by a research source.

Objective:
Participants will use direct and indirect measurement techniques to help determine the thickness of a paper towel. They will also apply area and volume formulas in this situation to find the thickness of one sheet. Using the research data for the thickness of one paper towel from this "big roll" brand, participants will determine the percent of error in their findings.

TExES Mathematics 4-8 Competencies. The beginning teacher:
III.008.A Selects and uses appropriate units of measurement (e.g., temperature, money, mass,
              weight, area, capacity, density, percents, speed, acceleration) to quantify, compare,
              and communicate information.
III.008.B Develops, justifies, and uses conversions within measurement systems.
III.010.A Uses and understands the development of formulas to find lengths, perimeters, areas,
              and volumes of basic geometric figures.
III.010.C Uses a variety of representations (e.g., numeric, verbal, graphic, symbolic) to analyze
              and solve problems involving two-and three-dimensional figures such as circles,
              triangles, polygons, cylinders, prisms, and spheres.
V.016.D   Communicates mathematical ideas using a variety of representations (e.g., numeric,
              verbal, graphic, pictorial, symbolic, concrete).

TEKS Mathematics Objectives: The student is expected to:
4.12   Measure to solve problems involving length, including perimeter, time, temperature,
               and area.
4.14.D   Use tools such as real objects, manipulatives, and technology to solve problems.
5.11.A   Measure to solve problems involving length (including perimeter), weight, capacity,
               time, temperature, and area.
5.14.D   Use tools such as real objects, manipulatives, and technology to solve problems.
6.6.C   Describe the relationship between radius, diameter, and circumference of a circle.
6.8.A   Estimate measurements and evaluate reasonableness of results.
6.8.B   Select and use appropriate units, tools, or formulas to measure and to solve problems
               involving length (including perimeter and circumference), area, time, temperature,
               capacity, and weight.
6.8.D   Convert measures within the same measurement system (customary and metric) based
               on relationships between units.
6.11.D   Select tools such as real objects, manipulatives, paper/pencil, and technology or
               techniques such as mental math, estimation, and number sense to solve problems.
7.9   Estimate measurements and solve application problems involving length (including
               perimeter and circumference), area, and volume.
7.13.D   Select tools such as real objects, manipulatives, paper/pencil, and technology or
               techniques such as mental math, estimation, and number sense to solve problems.
8.8.A   Find surface area of prisms and cylinders using concrete models and nets (two-
               dimensional models).
8.8.C   Estimate answers and use formulas to solve application problems involving surface
               area and volume.

8.14.D   Select tools such as real objects, manipulatives, paper/pencil, and technology or
               techniques such as mental math, estimation, and number sense to solve problems.

Terms.

Measurement, centimeter, millimeter, radius , diameter, circle, pi, area, thickness, dimension, volume of a cylinder, area of a circle, percent, percent of error , absolute error, relative error, precision, accuracy, significant digits

Materials.

- 1 roll of paper towels (thickness needs to be known) per group of 4
  Visit the website *http://cstl-csm.semo.edu / journet / GeneralStuff /Ptowel.htm*
  for information on brand-name paper towels and their thickness.
- Centimeter rulers
- Calculators
- Formulas (TAKS formula sheet for grade 8) - optional

Transparencies.

- *The Big Roll*

Activity Sheet(s)

- *The Big Roll*

References:

Guidelines for Mathematics and Mathematics Education Faculty (2004). Grades 4-8 Teacher
　　Tasks :Chapter 3 (p. 79).

Journet, Alan (2004). *Paper Towel Data: Spring 2004*. Southeast Missouri State University.
　　http://cstl-csm.semo.edu/journet/GeneralStuff/Ptowel.htm .

Procedure:

Steps	Questions/Math Notes
1. Have participants read the problem "*The Big Roll*" on Transparency #　. Ask them to examine the roll of paper towels on their table for dimensions, number of sheets per roll, etc. For more interest in the problem, provide different brands to the different groups. You will need to know the thickness of each brand of paper towel so that participants can determine the percent of error in their findings. A list of brand-name paper towels, their thickness, and other related facts (based on paper towel data provided online by Alan Journet , Department of Biology at Southeast Missouri State University) can be downloaded from the website provided in the materials list.	Ask questions to check participants' understanding of geometry concepts related to this problem. *What information did you get from the label of the paper towel roll that could be helpful in solving this problem?* *What are to trying to determine in this problem?* *What additional information do you need?* *What geometry/measurement concepts are involved in this problem?* *What measuring instrument will you use and why?*

2. Have participants work in groups of 4 to find the thickness of one sheet of their roll of paper towels. They are to take 20 measurements for the outer diameter and 20 for the inner diameter and use the average of each for the diameters of the cylinder and core. As each group completes their findings, provide the group with the thickness of their brand of paper towel from the "Paper Towel Data Spring 2004".	*Why did you select the unit(s) of measure for this problem?* *What is the greatest precision you can get with the measuring instrument selected?* *How many significant digits will you use to express your measurement? What is your reasoning?* *What solution strategy or strategies have you tried?* *What did you learn by using that strategy?* *What is another strategy that you could use?*
3. Debrief the activity and have different groups share their findings and strategies for solving the problem.	*How did you determine the measurements in your problem? What unit(s) did you use and why?* *How did you determine the absolute error? The relative error?* *How precise were your measurements? How do you know?* *How would you describe the accuracy in your results?* *What would you do differently based upon the discussions about measurement?* *How were your solution strategies alike? How were they different?* *What do you think contributed to your percent of error?* *How precise do you think these measurements need to be for the thickness of a paper towel? Explain your reasoning.* *Describe situations where more precision in measurement would be important.* *Why do you think it is important for students to think about precision and accuracy in measurement?*

Possible solution:
A "big roll" of Viva paper towels was used for this example. The roll contained 75 sheets with dimensions of 11" x 10.4" or 27.9 cm x 26.4 cm stated on the label. The total area was given as 11 m^2 or 119 sq ft on the wrapping. The thickness of one sheet (provided by Paper Towel Data Spring 2004 by Alan Journet of Southeast Missouri State University) is 0.537 mm obtained by using a micrometer caliper.

Measurements taken: Since the paper towel roll did not have a true cylindrical shape, 20 measurements were taken to the nearest one-tenth of a millimeter. Each measurement of the outer diameter has 4 significant digits and each measurement of the inner diameter has 3 significant digits as shown in the data below.

Outside diameter of roll in mm					Inside diameter of roll in mm				
140.0	142.2	142.8	143.0	138.8	45.0	46.0	44.7	43.5	44.6
137.5	139.0	140.1	138.9	138.0	44.0	45.0	44.8	44.0	44.9
139.0	141.8	142.8	139.5	137.0	44.5	45.6	44.5	43.8	44.0
139.6	140.7	139.0	138.1	141.7	44.0	44.7	44.0	44.5	44.0

mean diameter: 139.975 mm ≈ 140.0 mm   mean diameter: 44.5 mm
radius ≈ 69.9875 mm or 70 mm = 7 cm   radius ≈ 22.25 mm or 2.225 cm ≈ 2.2 cm

Find the volume of the entire cylindrical roll:
Measurements used: $r \approx 7$ cm , h = 27.9 cm (given)
$V_1 = \pi r^2 h$
$V_1 = \pi (7)^2 (27.9)$
$V_1 = 4{,}294.87$ cm^3 (rounded to the nearest one hundredth of a cubic centimeter)

Find the volume of the inner cylindrical roll:
Measurements used: $r \approx 2.2$ cm , h = 27.9 cm (given)
$V_2 = \pi r^2 h$
$V_2 = \pi (2.2)^2 (27.9)$
$V_2 = 424.23$ cm^3 (rounded to the nearest one hundredth of a cubic centimeter)

Subtract the volume of the inner cylindrical roll from the volume of the entire roll to find the volume of the paper towels on the roll.

$V_1 - V_2 = $ volume of paper towels on the roll
4,294.87 cm^3 - 424.23 cm^3 = volume of paper towels on the roll
3,870.64 cm^3 = volume of paper towels on the roll

Find the area of one paper towel:
Dimensions given: b = 26.4 cm , h = 27.9 cm
A = b h
A = 26.4 cm x 27.9 cm
A = 736.56 cm^2

Volume of the paper towels = area of one paper towel x number of towels x thickness (**t**)
                3,870.64 = 736.56 x 75 x **t**
            0.070067 cm = t

Convert this measurement to mm in order to compare to the given thickness of 0.537 mm.
0.070067 cm x 10 mm/ 1 cm = **0.70067 mm**
Therefore, **t = 0.700 mm** (approximate based upon measurements taken)

Find the absolute error: │(0.537-0.700)│ = 0.163

Find the relative error: relative error = (absolute error) ÷ (true value)
                = 0.163 ÷ 0.537
                = 0.304 (rounded to the nearest thousandth)

Find the percent of error: relative error expressed as a percent = 30.4%

The measurements were fairly consistent and seem to be close to the mean for both the inner and outer diameters of the cylindrical shapes of the roll. However, the measuring tool (centimeter ruler) is not as accurate as a calibrated caliper. The results reflect precision but not accuracy. The error reflects the degree of confidence in reporting the results.

Possible reasons for discrepancy in given thickness and measured/calculated thickness:
- Measuring device
- Measurement units selected
- Method of measuring
- Consistency in measurements
- Paper towel rolls can have a distorted cylindrical shape
- Averaging measurements of the diameter of the different cylinders

Extension.
Find the thickness of one paper towel from a different brand using what you learned from this experience. Compare your percent of error with the error obtained in the previous investigation.